JANUARY 1960

Journal
of
The Franklin Institute

1826

1960

Delay-Line Secondary Responses in AM and FM Sweep Integrators

HARRY URKOWITZ

A Servomechanisms Approach to Skill Development

EZRA S. KRENDEL AND DUANE T. MCRUER

Hydraulic Analogue for One Dimensional Unsteady Gas Dynamics

W. H. T. LOH

VOL. 269 NO. 1

NEW! TYPE K-3 UNIVERSAL POTENTIOMETER

makes Precise Voltage Measurements easier than ever before

- **Scale reads directly in voltage**
- **Guarded against humidity, static, leakage of any kind**
- **"Thermal Free" circuit reduces voltage errors**

You take your readings directly from a central reading window . . . no more mental arithmetic in adding up dial settings. You can use this unique instrument in hot, humid weather . . . in cold, dry weather . . . without worrying about leakage or static.

The many new design and construction features . . . metal case for electrostatic shielding . . . guarding of battery and detector circuits . . . enclosed switches and slidewires . . . thermal free circuit construction . . . central reading window . . . make this Type K-3 Universal (Guarded) Potentiometer the most advanced general purpose instrument available today. You will find it a worthy new addition to the Type K line . . . originated some 50 years ago . . . the world's most widely used precision potentiometers.

Six pages of data are required to give you full details. Ask for Data Sheet E-51 (6); write your nearest L&N Office or 4992 Stenton Avenue, Philadelphia 44, Pa.

LEEDS LN **NORTHRUP**

instruments *automatic controls* • *furnaces*

INDEX TO ADVERTISERS

	PAGE
Almo Radio Co.	x
Atlas Photo Engraving Co.	v
Bell Telephone Laboratories	viii
Biddle, James G., Co.	xi
Buten, M., & Sons	iv
Coleman, W. B., & Co.	ii
Damon & Foster	ii
Davidson, P. L.	ii
Fidelity-Philadelphia Trust Co.	xii
Franklin Institute, Laboratories	Fourth cover page
Harris-Dechant Associates	ii
Herbach & Rademan	v
Hess & Barker	x
Kearney Lumber Co.	v
Lancaster Press, Inc.	iii
Leeds & Northrup Co.	Second cover page
Leopold, Charles S.	ii
Levy & Co., Max	iv
Shanahan & Co.	v
Thomas Co., Arthur H.	iv
Wiljhelm, C.	x
Yarnall-Waring Co.	iii
Young, George, Co.	v

Second-class postage paid at Lancaster, Pa.

Consulting Engineers

HARRIS-DECHANT ASSOCIATES

FREDERICK H. DECHANT
Consulting Engineer

123 So. Broad St.
Philadelphia 9, Pa.

P. L. DAVIDSON
Consulting Engineer

Philadelphia, Pa. and
Greensboro, N. C.

DAMON & FOSTER

Consulting Engineers
Surveyors

CHESTER PIKE & HIGH ST.
SHARON HILL, PA.

CHARLES S. LEOPOLD, INC.
ENGINEERS

215 SOUTH BROAD ST.
PHILA. 7, PA.

W. B. COLEMAN & CO.
Metallurgists - Chemists - Engineers

Spectrographic and Spectrophotometric Analysis
Chemical and Physical Testing—Microscopic Investigation
Huey and Salt Spray Corrosion Tests—Boiler Water Conditioning

9TH & RISING SUN AVE., PHILADELPHIA 40, PA.

YARWAY

STEAM PLANT EQUIPMENT

OVER 50 YEARS A PARTNER
IN THE PROGRESS OF
STEAM ENGINEERING

Yarnall-Waring Company • Philadelphia 18

A COMPLETE PRINTING SERVICE

GOOD PRINTING does not just happen; it is the result of careful planning. The knowledge of our craftsmen, who for many years have been handling details of composition, printing and binding, is at your disposal. For over seventy-five years we have been printers of scientific and technical journals, books, theses, dissertations and works in foreign languages. Consult us about your next job.

. PRINTERS OF .
JOURNAL OF THE
FRANKLIN INSTITUTE

LANCASTER PRESS, Inc.

PRINTERS • BINDERS • ELECTROTYPERS

ESTABLISHED 1877 LANCASTER, PA.

PRECISION RULINGS ON GLASS

Scales . Grids . Reticles
Halftone Screens

MAX LEVY & CO. . Wayne Ave. & Berkley St. Phila., Pa.

EVERYTHING IN PAINTS

BUTEN'S

PAINT STORES

Phila., Chester, Reading, Upper Darby, Bryn Mawr, Darby, Doylestown
Camden, N. J. — Wilmington, Del.

ARTHUR H. THOMAS CO.

Laboratory Apparatus and Reagents

Selected for Chemistry and Biology

Adequate stocks of 16,000 Apparatus items and 6,000 Reagent items carried in our Philadelphia warehouse for immediate shipment.

VINE STREET AT THIRD PHILADELPHIA 5, PA.

Commercial Stationery
Loose Leaf—Blank Books
Filing Equipment
Office Supplies

SHANAHAN & CO.
22 S. 18 St. Ri 6-0333

IS THE ANSWER
ATLAS PHOTO ENGRAVING CO. 1208 CHERRY ST. PHILA., 7, PA.

GEORGE YOUNG COMPANY
RIGGING AND HAULING
CONTRACTORS
S. W. COR. 20th & OREGON AVE.
PHILADELPHIA 45, PA. HO 7-5315

HEADQUARTERS
FOR
RADIO PARTS ELECTRONIC COMPONENTS
GEIGER-MÜLLER COUNTER TUBES
AND
RADIO ACTIVITY DETECTORS
HERBACH & RADEMAN
1204 ARCH STREET PHILADELPHIA 7, PA.

KEARNEY LUMBER COMPANY
Lumber of every description for every purpose
2916–26 Kensington Ave.
GA 6-5720 Phila., Pa.
"Our Fleet of Trucks Deliver Anywhere"

AWARDS BY THE INSTITUTE

The Franklin Medal (1914—Gold Medal).—This medal is awarded annually to those workers in physical science or technology, without regard to country, whose efforts, in the opinion of the Institute, acting through its Committee on Science and the Arts, have done most to advance a knowledge of physical science or its applications.

The Elliott Cresson Medal (1848—Gold Medal).—This medal is awarded for discovery or original research, adding to the sum of human knowledge, irrespective of commercial value; leading and practical utilizations of discovery; and invention, methods or products embodying substantial elements of leadership in their respective classes, or unusual skill or perfectior in workmanship.

The Howard N. Potts Medal (1906—Gold Medal).—This medal is awarded for distinguished work in science or the arts; important development of previous basic discoveries, inventions or products of superior excellence or utilizing important principles.

The John Price Wetherill Medal (1925—Silver Medal).—This medal is awarded for discovery or invention in the physical sciences or for new and important combinations of principles or methods already known.

The Edward Longstreth Medal (1890—Silver Medal).—This medal is awarded for inventions of high order and for particularly meritorious improvements and developments in machines and mechanical processes. In the event of an accumulation of the fund for medals beyond the sum of one hundred dollars, it is competent for the Committee on Science and the Arts to offer from such surplus a money premium for some special work on any mechanical or scientific subject that is considered of sufficient importance.

The Louis E. Levy Medal (1923—Gold Medal).—This medal is awarded to the author of a paper of especial merit, published in the JOURNAL OF THE FRANKLIN INSTITUTE, preference being given to one describing the author's experimental and theoretical researches in a subject of fundamental importance.

The George R. Henderson Medal (1924—Gold Medal).—This medal is awarded for meritorious inventions or discoveries in the field of Railway Engineering.

The Walton Clark Medal (1926—Gold Medal).—This medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."

The Frank P. Brown Medal (1938—Silver Medal).—This medal is awarded to inventors for discoveries and inventions involving meritorious improvements in the building and allied industries.

The Newcomen Medal (1943—Gold Medal).—This medal is awarded, not oftener than once in three years, for achievement in the field of Steam.

The Francis J. Clamer Medal (1943—Silver Medal).—This medal is awarded at least once in five years for meritorious achievement in the field of Metallurgy.

The Stuart Ballantine Medal (1946—Gold Medal).—This medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

The Boyden Premium (1859).—This premium is awarded not oftener than once in five years to any resident of North America who has recently made a notable experimental determination of the speed, in free space, of radiation in any region of the entire spectrum.

The William M. Vermilye Medal (1937—Bronze Medal).—This medal is awarded not oftener than biennially in recognition of outstanding contribution in the field of Industrial Management.

The Certificate of Merit (1882).—A Certificate of Merit is awarded to persons adjudged worthy thereof for meritorious inventions, discoveries or improvements in physical processes or devices.

For further information relating to these awards apply to the Secretary, Committee on Science and the Arts

THE FRANKLIN INSTITUTE

OFFICERS

President.....WYNN LAURENCE LEPAGE

Secretary.....W. F. JACKSON, JR.

Treasurer.....C. M. WATERBURY

Director of Laboratories.....NICOL H. SMITH

Assistant Secretaries.....WALTER A. R. PERTUCH, C. M. WATERBURY

BOARD OF MANAGERS

Term expires in 1960	*Term expires in 1961*	*Term expires in 1962*
BRANDON BARRINGER	HENRY B. BRYANS	HENRY B. ALLEN
EDWARD G. BUDD, JR.	HENRY M. CHANCE II	G. H. CLAMER
J. FRANK COX	FRANCIS J. CHESTERMAN	A. FELIX DUPONT, JR.
JOSEPH GRAY JACKSON	JAMES CREESE	J. G. RICHARD HECKSCHER
JAMES MCGOWAN, JR.	RUPEN EKSERGIAN	C. L. JORDAN
RICHARD T. NALLE	JOSEPH A. FISHER	LIONEL F. LEVY
S. WYMAN ROLPH	MORTON GIBBONS-NEFF	R. G. RINCLIFFE
JOHN RUSSELL, JR.	GAYLORD P. HARNWELL	JAMES H. ROBINS
W. MAXWELL SCOTT, JR.	WYNN LAURENCE LEPAGE	PHILIP H. WARD, JR.
I. MELVILLE STEIN	EMERY W. LOOMIS	HARLESTON R. WOOD

GEORGE WHARTON PEPPER, Honorary Member of the Board

EXECUTIVE COMMITTEE OF THE BOARD OF MANAGERS

S. WYMAN ROLPH, *Chairman*
RICHARD T. NALLE, *Vice Chairman*
HENRY B. BRYANS
WYNN LAURENCE LEPAGE

STANDING COMMITTEES OF THE BOARD OF MANAGERS

Nominating Committee.....RICHARD T. NALLE, *Chairman*
Budget Committee.....HENRY B. BRYANS, *Chairman*
Personnel Policy Committee.....J. G. RICHARD HECKSCHER, *Chairman*
Buildings and Grounds Committee.....W. MAXWELL SCOTT, JR., *Chairman*
Education Committee.....JAMES CREESE, *Chairman*
Research and Development Committee.....G. H. CLAMER, *Honorary Chairman*; S. WYMAN ROLPH, *Chairman*
Bartol Research Foundation Committee.....G. H. CLAMER, *Honorary Chairman*; GAYLORD P. HARNWELL, *Chairman*
Biochemical Research Foundation Committee.....A. FELIX DUPONT, JR., *Chairman*
Educational Radio and Television Committee.....HENRY B. BRYANS, *Chairman*

GENERAL COMMITTEES OF THE INSTITUTE

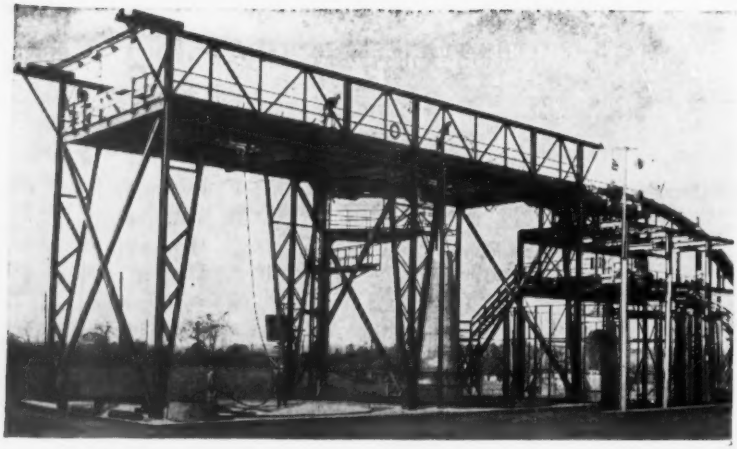
Committee on Science and the Arts.....WILLIAM G. SCHMIDT, *Chairman*
Membership Committee.....MORTON GIBBONS-NEFF, *Chairman*
Investment Committee.....BRANDON BARRINGER, *Chairman*
Endowment Committee.....MORTON GIBBONS-NEFF, *Chairman*

ADVISORY COMMITTEES OF THE INSTITUTE

Museum and Memorial Committee.....I. MELVILLE STEIN, *Chairman*
Library Committee.....PENROSE R. HOOPES, *Chairman*
Meetings Committee.....C. L. JORDAN, *Chairman*
Public Relations Committee.....C. L. JORDAN, *Chairman*
Publications Committee.....LIONEL F. LEVY, *Chairman*
Laboratories Committee.....RUPEN EKSERGIAN, *Chairman*

HOSTESS COMMITTEE OF THE INSTITUTE

MRS. ISAAC A. PENNYPACKER, *Chairman*



SHIP WITHOUT AN OCEAN

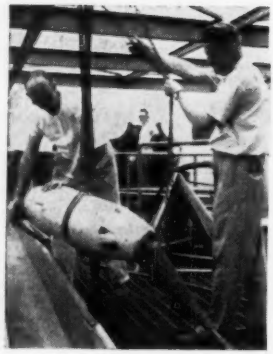
How do you lay a cable on the ocean floor—a cable that is connected to scores of large, heavy amplifiers? How do you "overboard" such a system in a continuous operation, without once halting the cable ship?

Bell Telephone Laboratories engineers must answer these questions in order to lay a new deep-sea telephone system designed to carry many more simultaneous conversations. They're experimenting on dry land because it is easier and more economical than on a ship. Ideas that couldn't even be attempted at sea are safely tested and evaluated.

In one experiment, they use a mock-up of the storage tank area of a cable ship (above). Here, they learn how amplifiers (see photo right), too rigid and heavy to be stored with the cable coils *below* decks, must be positioned *on* deck for trouble-free handling and overboarding.

Elsewhere in the Laboratories, engineers learn how best to grip the cable and control its speed, what happens as the cable with its amplifiers falls through the sea, and how fast it must be payed out to snugly fit the ocean floor. Oceanographic studies reveal the hills and valleys which will be encountered. Studies with naval architects show how the findings can be best put to work in actual cable ships.

This work is typical of the research and development effort that goes on at Bell Labs to bring you more and better communications services.



Experimental amplifier about to be "launched" from "cable ship." Like a giant string of beads, amplifiers and connecting cable must be overboarded without stopping the ship.

BELL TELEPHONE LABORATORIES

WORLD CENTER OF COMMUNICATIONS RESEARCH AND DEVELOPMENT

Journal
of
The Franklin Institute

MANAGING EDITOR, NANCY S. GLENN
Editor Emeritus, HENRY B. ALLEN

Associate Editors:

CHARLES B. BAZZONI
WILLIAM H. BOGHOSIAN
CHARLES L. CRITCHFIELD
RUPEN EKSERGIAN
VIRGIL MORING FAIRES
PAUL D. FOOTE
WALTER C. JOHNSON
ROBERT S. LIVINGSTON
C. E. K. MEES
F. J. MURRAY
W. F. G. SWANN
HUGH S. TAYLOR
MERLE A. TUVE

ARTHUR L. DAY, *Emeritus*

Committee on Publications:

LIONEL F. LEVY, *Chairman*
HENRY B. ALLEN
JOHN S. BURLEW
CLARENCE L. JORDAN
PHILIP H. WARD, JR.

Vol. 269 **JANUARY, 1960** **No. 1**

CONTENTS

Delay-Line Secondary Responses in AM and FM Sweep Integrators ..... HARRY URKOWITZ 1

A Servomechanisms Approach to Skill Development ..... EZRA S. KRENDEL AND DUANE T. MCRUER 24

Hydraulic Analogue for One Dimensional Unsteady Gas Dynamics ..... W. H. T. LOH 43

The Franklin Institute

Minutes of the Stated Meeting, December 16, 1959 56

Committee on Science and the Arts 57

Membership 57

Library 58

Franklin Institute Laboratories for Research and Development 60

Book Reviews 63

Book Notes 68

Publications Received 70

Current Topics 71

Published monthly by The Franklin Institute of the State of Pennsylvania, Benjamin Franklin Parkway at Twentieth St., Philadelphia 3, Pennsylvania. Printed by Lancaster Press, Inc., Prince and Lemon Sts., Lancaster, Pennsylvania.

Subscription rates: Domestic, $10.00 a year; Foreign, $11.00 a year. Single current issues, $1.00. Reprints of papers published prior to July, 1959, 50 cents per installment. After this date, the publishing of reprints will be discontinued. All inquiries regarding subscriptions, change of address, sale of back issues and reprints, should be addressed to the Philadelphia office.

Indexes to the semi-annual volumes of the JOURNAL are published in the June and December numbers.

Copyright © 1960, by The Franklin Institute of the State of Pennsylvania.

C. WILJHELM
INDUSTRIAL INSTRUMENTS, INC.
HUNTINGDON VALLEY, PA.
CApital 4-2043

SALE OR RENTAL—PYROMETERS, FLOWMETERS, THERMOMETERS, CO_2 RECORDERS, MOST OTHER TYPES OF INDUSTRIAL INSTRUMENTS & CONTROLS

REPAIR—Competent & Quick Repair of *ALL* Types of Industrial Instruments —at *your* plant or in our laboratory

Almo RADIO CO.
EXECUTIVE OFFICES 913 ARCH ST. PHILA., PA. · WA 2-5918
10 LOCATIONS IN FOUR STATES TO SERVE YOU

DOWNTOWN PHILA.	WEST PHILA.	NORTHEAST	INDUSTRIAL DEPT.
913 ARCH STREET	6205 MARKET ST.	7540 FRANKFORD AVE.	412 N. 6th STREET
WA 2-5918	GR 6-1706	DE 2-2234	WA 2-5918

BRANCHES: CAMDEN, N. J. • ATLANTIC CITY, N. J. • TRENTON, N. J.
NORRISTOWN, PENNA. • WILMINGTON, DELAWARE • SALISBURY, MD.

60 YEARS OF EXPERIENCE
ENGINEERING MACHINISTS
DESIGN · MANUFACTURE · REPAIRS
HESS & BARKER
930 Washington Ave. • HOward 7-1121 • Phila. 47, Pa.

BIDDLE
Instruments

and miscellaneous laboratory equipment cover a wide field of electrical, mechanical and scientific testing, measuring and experimental work.

For a complete file of all our products write for Bulletin 19-53

JAMES G. BIDDLE CO.
Electrical & Scientific Instruments
1316 Arch Street—Philadelphia 7, Pa.

He's set for college . . .

we're saving at

FIDELITY
PHILADELPHIA TRUST COMPANY

26 offices in Philadelphia, Bucks, Delaware, and Montgomery Counties

Member Federal Deposit Insurance Corporation

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 269 JANUARY, 1960 No. 1

DELAY-LINE SECONDARY RESPONSES IN AM AND FM SWEEP INTEGRATORS *

BY

HARRY URKOWITZ [1]

ABSTRACT

A video sweep integrator is a device for adding the successive detected radar returns of successive transmitted pulses. One type of video sweep integrator uses an ultrasonic delay line for storage and the addition is obtained by means of a closed regenerative loop operating continuously with exponential memory. Delay-line secondary, or spurious, responses result from multipath propagation through the delay line. The cumulative effect of these responses after they are recirculated in the sweep integrator may be very significant. The exact effect depends to a certain extent upon the type of modulation that is propagated through the delay line. In this paper the buildup effects of secondaries are determined for both AM and FM transmission through the delay line. With AM there is coherent buildup, which results in a secondary buildup relatively much greater than the buildup of the desired signal. With frequency modulation it is possible to alleviate the buildup effect of secondaries by introducing a small amount of hum deviation on the carrier frequency, which results in a random buildup. Experiments with FM sweep integrators confirm the expected coherent buildup without hum deviation and also confirm that random buildup occurs when the hum deviation is introduced.

This paper also derives formulas for the effects of delay-line secondaries when the integrator operates with uniform or constant memory for a finite time. In this mode of operation with FM transmission through the delay line, the secondaries build up essentially noncoherently without the introduction of hum deviation.

I. INTRODUCTION

A complete analysis of the effect of delay-line secondary responses is highly involved mathematically. Rather than burden the reader with all these details they are relegated to the Appendix. The results of the

* The work presented in this paper is based upon some work done by Martin Axelbank of MIT's Lincoln Laboratory, who derived, among other things, the limiting value of distortion caused by delay-line secondaries in an FM sweep integrator. Lincoln Laboratory has given permission for the work to be declassified.

[1] Research Division, Philco Corporation, Philadelphia, Pa.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

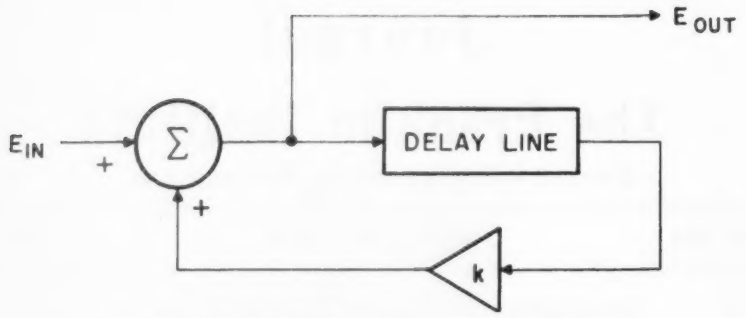


FIG. 1. Symbolic block diagram of a video sweep integrator.

analysis are presented in the body of this paper by means of physical arguments and appeals to plausibility. Otherwise, the reader might become bogged down in mathematical formulas.

A video sweep integrator is a device for adding successive radar video traces. It consists, essentially, of a closed loop containing a delay or storage medium, typically an ultrasonic delay line. A symbolic block diagram is shown in Fig. 1. The magnitude of the loop gain, or feedback factor, is k, which is less than unity if the sweep integrator is operated continuously. In an alternative mode of operation, called "iteration," a number of successive traces are added with unity feedback factor; the sum is then read out and the intelligence in the loop wiped out. The process is then repeated.

In an ultrasonic delay line, the video must be modulated upon a carrier wave before being applied to the delay line. After integration, the carrier must be demodulated. If a train of identical pulses, each of unit height and uniformly spaced with a period equal to the loop delay, is the input to the sweep integrator, the amplitudes of successive outputs will be as shown in Eq. 1.

$$\left.\begin{aligned} &1: 1 \\ &2: 1 + k \\ &3: 1 + k + k^2 \\ &4: 1 + k + k^2 + k^3 \\ &\cdots \quad \cdots \\ &n: 1 + k + k^2 + k^3 + \cdots + k^{n-1} = \frac{1 - k^n}{1 - k} \\ &\cdots \quad \cdots \\ &\infty : \frac{1}{1 - k} \end{aligned}\right\} \tag{1}$$

The successive outputs of an iterator whose input is that given above are:

$$\begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ n \end{matrix}$$

In this paper, our interest is not in the noise-reduction capabilities of sweep integrators; this aspect has been treated elsewhere (1–4).[2] Our interest is in the secondary responses of the delay line and their buildup compared to the buildup of the inserted signal.

II. DESCRIPTION OF SECONDARIES

Spurious or secondary responses are caused by multipath propagation through the delay line. A simplified representation of the delay line consists of several transmission paths in parallel with their outputs connected together. If the delay-line input signal is $e_1(t)$, the output will be given by Eq. 2:

$$e_0(t) = \sum_{j \neq 0} C_j e_1(t - \tau_j) = C_0 e_1(t - \tau_0) + \sum_{j \neq 0} C_j e_1(t - \tau_j) \qquad (2)$$

where

C_0 = relative transmission of main path
τ_0 = time delay of main path
C_j = relative transmission of j^{th} secondary path
τ_j = time delay of j^{th} secondary path.

It has been assumed that the bandwidth of the delay line is great enough so that each $e_1(t - \tau_j)$ has the same shape as $e_1(t)$. We may get a picture of the spurious responses by letting $e_1(t)$ be a train of

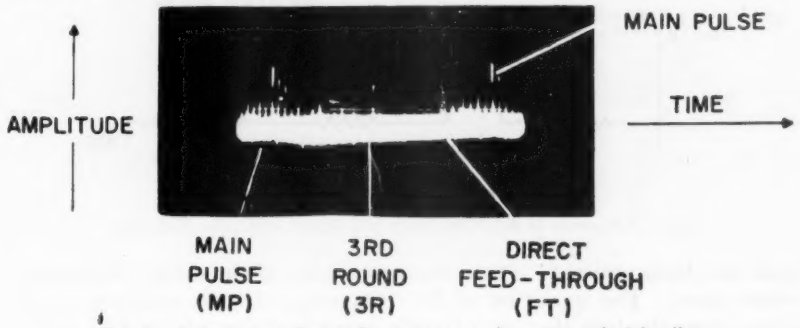
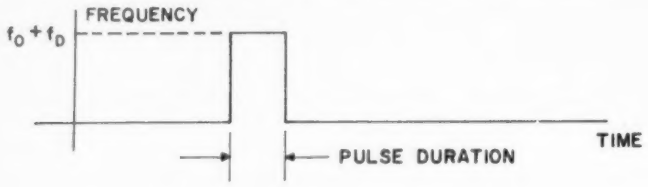


FIG. 2. Appearances of secondary responses of an ultrasonic delay line with RF pulse at input.

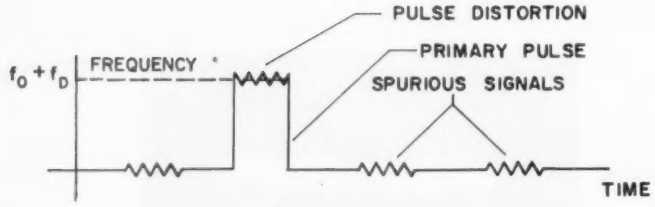
[2] The boldface numbers in parentheses refer to the references appended to this paper.

carrier (RF) pulses with a repetition period greater than three times the delay of the delay line. The envelope of the delay-line output will show the primary delayed pulse and a number of secondary pulses. A photograph of an oscilloscope screen with the delay-line output applied to the vertical deflection plates is shown in Fig. 2. The third-round pulse is sometimes called "triple travel." The feedthrough, or leakage, is the pulse which appears at the output with substantially zero delay. Note the presence of fluctuation noise. In the material to follow it will usually be assumed that the pulse duration is small enough so that there is no overlap of secondary pulses.

Now let us suppose that a frequency-modulated pulse is applied to the delay line. By this, I mean that a carrier wave of constant amplitude is applied to the delay line and that its frequency is constant at f_0 except for a time equal to the pulse duration, during which time it is $f_0 + f_D$. This is illustrated in Fig. 3*a*. The frequency of the delay-line output is shown in Fig. 3*b*. In these figures the fluctuation



(a) Frequency of Delay Line Input



(b) Frequency of Delay Line Output

FIG. 3. Frequency of delay-line input and output with pulse FM input.

noise has been omitted. Note the appearance of the output frequency *versus* time. The variation of the frequency of each spurious signal arises from the fact that at a time τ_j after insertion of the FM pulse, there exist two waves: one is the carrier wave of frequency f_0 and amplitude C_0; the other is the j^{th} delayed secondary which has an

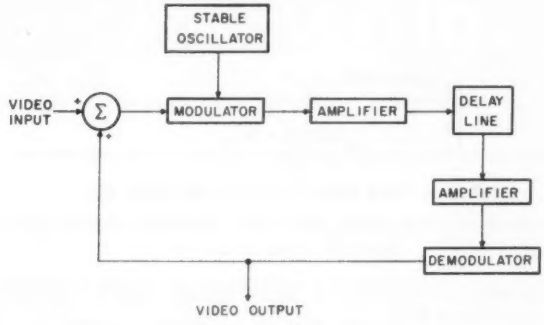
amplitude C_j and a frequency $f_0 + f_D$ for a time equal to the pulse duration. The frequency of the resultant shows a time variation; the *frequency* of this time variation (ripple frequency) is equal to f_D and the *amplitude* of this time variation (ripple amplitude) is given by $f_D C_j / C_0$. The relative phase of this spurious ripple is φ_j as indicated in Eq. 3:

$$\text{Frequency of } j^{\text{th}} \text{ spurious signal} = \frac{C_j}{C_0} \cos (2\pi f_D t - \varphi_j) \qquad (3)$$

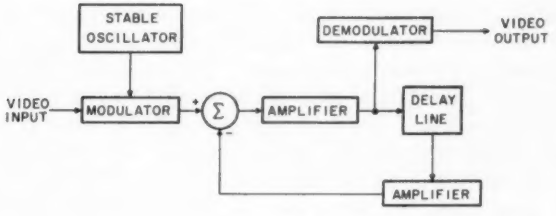
where $\varphi_j = 2\pi f_D \tau_j + 2\pi f_0 (\tau_j - \tau_0)$.

Note that φ_j depends upon the deviation, the carrier frequency, and the time delay of the j^{th} spurious path.

The distortion on top of the main delayed pulse is the result of all the secondary signals acting together. The maximum distortion cannot occur unless the various φ_j differ by multiples of 2π. Conceivably,



(a) Video Addition



(b) RF Addition

FIG. 4. Two embodiments of an AM sweep integrator.

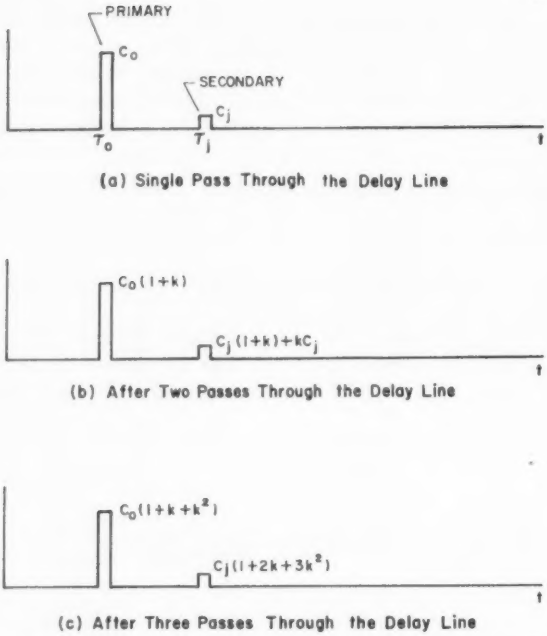


FIG. 5. Primary and secondary pulses after passage through the delay line in an AM sweep integrator.

there exists a certain pattern of secondaries which will produce this effect when combined with the correct values of f_D and f_0. It is more plausible to suppose that the phases are randomly distributed. Then the resultant distortion amplitude is a statistical quantity which depends upon the particular pattern of secondaries. More will be said about this later.

III. AM INTEGRATION AND ITERATION

Two simple embodiments of an AM sweep integrator are shown in Fig. 4. In one embodiment, the signal is modulated upon the carrier and demodulated after each circulation. In the other, the modulation and demodulation operations take place outside the loop, so that addition at RF is necessary. The loop gain or feedback factor is determined by the loop amplifiers; and by proper gating, the integrator may be made into an iterator. It is important to note that the total delay around the loop must be very close to the repetition interval of the input pulse train.

Let us concentrate on the main signal and a single secondary. When a single RF pulse is applied to the delay line, the envelope of the output will appear as in Fig. 5*a*, and the ratio of secondary height to

primary height will be C_j/C_0. Now, both primary and secondary are multiplied by k and the first primary is added to the next primary in the incoming pulse train, giving rise to a pulse of amplitude $C_0(1 + k)$. This gives rise to a secondary of amplitude $C_j(1 + k)$ which adds to the previous secondary whose amplitude is now kC_j, so that the total secondary amplitude is, after two pulses have entered the delay line, $C_j(1 + 2k)$. This situation is illustrated in Fig. 5*b*. The next time around, the primary signal amplitude becomes $C_0(1 + k + k^2)$ which gives rise to a secondary of amplitude $C_j(1 + k + k^2)$; this adds to k times the previous secondary so that, after three pulses have entered the delay line, the secondary amplitude is $C_j(1 + 2k + 3k^2)$. This is illustrated in Fig. 5*c*. After n pulses have entered the delay line, the amplitude of the primary pulse is given by $\frac{C_0(1 - k^n)}{1 - k}$. However, the amplitude of the secondary pulse will be:

$$C_j(1 + 2k + 3k^2 + 4k^3 + \cdots + nk^{n-1}) = C_j\left[\frac{1 - k^n}{(1 - k)^2} - \frac{nk^n}{1 - k}\right]. \quad (4)$$

Therefore, the built-up ratio of secondary-to-primary is:

$$\frac{C_j}{C_0}\left(\frac{1 - k^n}{1 - k} - nk^n\right). \quad (5)$$

In the limit of an infinitely long pulse train, this becomes:

$$\text{Ratio of built-up secondary-to-primary} = \frac{C_j}{C_0(1 - k)}. \quad (6)$$

It can be seen that the secondary pulse builds up more rapidly than the primary pulse. Quantitatively, the secondary buildup, measured in decibels, is twice the primary buildup, so that if the single-pass primary-to-secondary ratio is 40 db, and if the signal buildup is 20 db, the secondary buildup will be 40 db, resulting in a built-up primary-to-secondary ratio of 0 db. Thus, the secondaries may present a severe limitation on the dynamic range of the sweep integrator, more severe than fluctuation noise presents.

The "coherent" buildup of secondaries is a direct consequence of the way in which addition takes place in an AM sweep integrator, whether envelope or carrier addition is used. There appears to be little that can be done to alleviate the situation with AM except by using very high quality delay lines.

In the case of iteration, where $k = 1$ for a finite time, the built-up primary will be nC_0 and the built-up secondary will be

$$C_j(1 + 2 + 3 + \cdots + n) = \frac{n(n + 1)C_j}{2}. \quad (7)$$

Then:

$$\text{Built-up secondary-to-primary ratio} = \frac{C_i(n+1)}{2C_0}. \tag{8}$$

As a numerical example, for $n = 10$, the primary will build up 20 db, while the secondary will build up approximately 35 db, so that the integrated primary-to-secondary ratio will be 15 db worse than the unintegrated primary-to-secondary ratio.

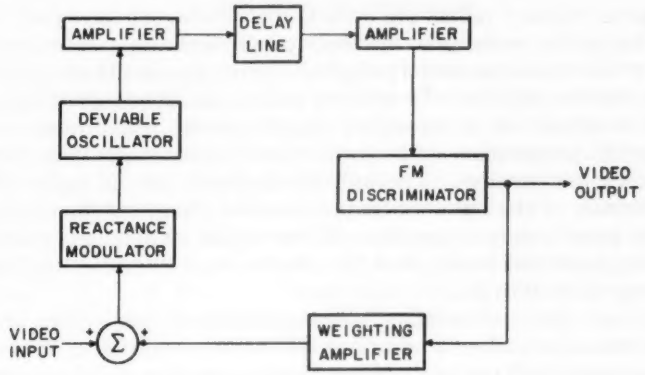
IV. FM SWEEP INTEGRATION (5, 6) AND ITERATION

A simple embodiment of a sweep integrator using FM transmission through the delay line is shown in Fig. 6*a*. In this embodiment, the signal is modulated and demodulated after each circulation around the loop, and the only difference between this and the AM sweep integrator of Fig. 5*a* is in the modulation going through the delay line. A more sophisticated embodiment is shown in Fig. 6*b*. Here there are two feedback paths for the signal; one of these paths is regenerative and is completely at RF. The successive circulating signals, consisting of frequency deviations on a carrier, are added directly in mixer X. The purpose of mixer Y is to translate the carrier frequency upward so that the frequency entering the delay line will be the proper one after it has been heterodyned with the output of the deviable oscillator. Since frequency deviations add directly, the regenerative loop has an effective feedback factor of unity. To get a feedback factor less than unity, a small amount of degeneration is introduced at video by means of an auxiliary path from output to input. This degenerative weighting is $1 - k$, where k is the over-all feedback factor.

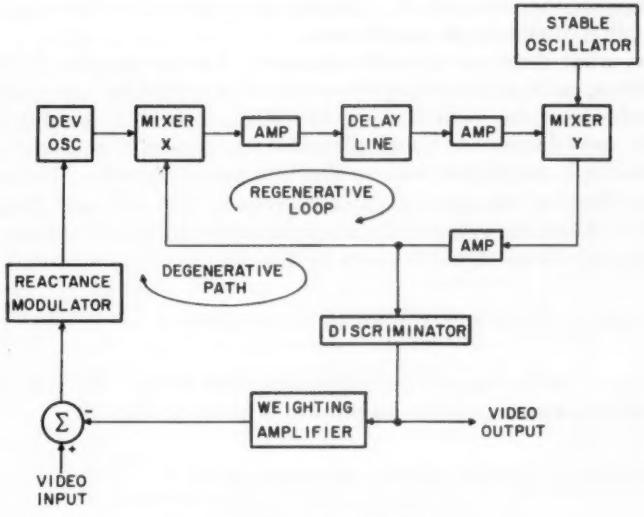
The embodiment of Fig. 6*b* may be made to operate as an iterator by imposing the proper gating on the weighting amplifier, or by other methods which are of no interest here.

V. SECONDARIES IN FM ITERATION (UNITY WGEIHTING)

Mathematical details for the cases of FM iteration and integration are given in the Appendix. Here we consider the problem without mathematics, summarizing the results. When the delay line is used in an FM sweep integrator or an iterator, successive inputs are added; the secondary responses are also added, but these are built up to a greater relative extent than the primary signal. Now, let us trace through the addition of successive pulses with unity weighting. As the first pulse deviation arrives at the delay-line output, it is accompanied by its pattern of secondary ripples, the largest one having a transmission C_M. This is the one upon which we concentrate. The next pulse in the input pulse train causes a deviation which adds directly to the deviation of the first pulse; the result is a pulse deviation twice as large



(a) Video Addition



(d) Frequency Addition

FIG. 6. Two forms of the FM sweep integrator.

as before. This larger deviation produces a secondary ripple with amplitude proportional to the second pulse deviation and, therefore, twice as large as the ripple amplitude from the first pulse. This second ripple occurs at the same time as the first ripple. The resultant amplitude depends upon the phases of the two secondary ripples. As

successive primary pulses are added, the number of secondary ripples occurring at the same time increases proportionately. The amplitude of the r^{th} successive secondary ripple is r times the amplitude of the first. Thus, after the addition of n primary pulses, the largest secondary ripple is the resultant of n secondary ripples whose amplitudes form an arithmetic progression and whose ripple frequencies also form an arithmetic progression. The individual phases are all quite different and, because of the different frequencies and phases of the ripples, it is not too great a step to say that all the ripple components combine in random phase and to say that the phases are uniformly distributed in the range from 0 to 2π.

We see that determining the magnitude of the ripple from the largest secondary after n additions has become a problem in statistics, and we must find the probability density function for the secondary-ripple resultant. If the number of additions, n, is large, the density function for the resultant is approximately a Rayleigh density function, with the modification that the maximum possible value for the resultant amplitude is the simple sum of the component amplitudes. The important parameter for us is R_0, the rms sum, defined as the root-mean-square value of the ripple amplitudes.

Now, what does one actually observe? Let us imagine that complete traces, each representing the result of n additions, are applied to a cathode-ray tube, which may be either deflection modulated or intensity modulated. It seems plausible to suppose that the observed spurious ripple amplitude will be the average amplitude. For a Rayleigh distribution, the average value is given by $R_0\sqrt{\pi/2}$, or 2 db greater than R_0. When the value of R_0 is computed for large n, it is found that the largest spurious signal is given by Eq. 9:

$$\text{Average relative maximum spurious signal} \approx \frac{\sqrt{\pi/2}\ C_M}{C_0}\sqrt{\frac{n}{3}} \qquad (9)$$

where C_M/C_0 is the largest secondary response ratio. By way of contrast, the maximum possible amplitude is given by Eq. 10:

$$\text{Maximum possible relative spurious signal} = \frac{n(n+1)C_M}{2C_0}. \qquad (10)$$

The distortion builds up in a way similar to that of the spurious signals. Applying the same techniques, it is found that the average relative distortion is given by Eq. 11 for large n:

$$\text{Average relative distortion} = \sqrt{\frac{\pi n}{6C_0}\sum_{j\neq 0} C_j^2}. \qquad (11)$$

Note that all the secondary signals contribute.

VI. CONTINUOUS FM INTEGRATION

Now let us turn to continuous sweep integration. In this process we imagine an infinite train of input pulses being applied. After a time, the total deviation builds up to $1/(1-k)$ times the deviation of a single pulse. Thus, the pulse deviation being circulated now is a constant, and the secondary ripple caused by each circulation adds to the secondary ripple of the previous trace whose amplitude is reduced to k (the feedback factor) times its original amplitude. Although the signal buildup has reached its maximum, the secondary ripples continue to pile up on top of each other. The amplitudes form a geometric progression: $1, k, k^2, k^3, \cdots$. These individual ripples all occur at the same time and the resultant amplitude depends upon their phases. Their individual phases, in turn, depend on the carrier frequency, the deviation, and the time of the secondary after the primary pulse. Since the primary deviation has become constant, if the carrier frequency also remains constant, it can be expected that all the ripple components will be in phase and the resultant will be the sum of the individual amplitudes. Thus the buildup factor for the secondary ripple is $1/(1-k)$ times the buildup factor for the primary signal. Such a buildup is called coherent buildup, in contrast to the random buildup occurring when the ripple phases are random.

Coherent buildup can be quite serious and can put a severe limitation on the dynamic range of the integrator. One wonders what might cause random phasing and, therefore, random buildup. The only quantity which might be varied is the carrier frequency. This can vary naturally because of natural hum appearing at the reactance tube controlling the deviable oscillator, or because of hum deliberately introduced for the purpose of causing random phasing of the secondary ripples. If random phasing can be accomplished, what will be observed will be the average amplitude given by $R_0\sqrt{\pi/2}$, where R_0 is the rms sum of the ripple amplitudes. Under this condition, the largest spurious signal has an average amplitude given by Eq. 12:

$$\text{Average relative maximum spurious signal} = \frac{\sqrt{\pi/2}\, C_M}{C_0\sqrt{1-k^2}}. \qquad (12)$$

Table I shows the relative buildup of delay-line secondaries as a function of feedback factor for maximum or coherent buildup and for random buildup. Note the very large differences at the higher feedback factors.

The total distortion appearing on each primary pulse builds up in the same way as the spurious ripples. Thus, for random buildup, the average relative distortion is given by Eq. 13.

$$\text{Average relative distortion} = \frac{\sqrt{\pi/2}}{C_0}\sqrt{\frac{\sum_{j\neq 0} C_j^2}{1-k^2}}. \qquad (13)$$

TABLE I.—*Relative Buildup of Delay-Line Secondaries in an FM Sweep Integrator.*

Feedback Factor, k	Relative Buildup	
	Maximum (coherent), db	Average, db
0.99	40	19
0.98	34	16
0.97	30.5	14.2
0.96	28	13
0.95	26	12
0.93	23	10.7
0.90	20	9.2
0.85	16.5	7.5

Note the presence of the rms sum of all the delay-line secondary responses.

VII. EXPERIMENTAL RESULTS

What about experimental verification of all the foregoing analysis? Experiments with three separate FM sweep integrators are described below. In all cases, the sweep integrator output was observed on a deflection-modulated oscilloscope (an "A" scope trace). In two of the experiments a.c. was used on all filaments. The results can be briefly summarized as follows: the observed amplitudes after buildup were those predicted on the assumption of random buildup. In the third sweep integrator, d.c. was applied to all filaments and the observed amplitude of spurious signal after buildup was that predicted on the basis of coherent buildup. Then a small amount of hum deviation was impressed upon the deviable oscillator. The observed amplitudes after buildup became those predicted on the basis of random buildup.

A. FM Sweep Integrator Using a 1000-Microsecond Delay Line

This sweep integrator was the first to use FM transmission through the delay line. It was constructed under M.I.T.'s Lincoln Laboratory Subcontract 16, Prime Contract AF-19(122)-458. The line itself was equipped with 30-mc transducers. When its properties were measured, the largest secondary response was the only one measured. Its amplitude relative to the main signal was -35 db $\pm$ 3 db. The input to the sweep integrator was an infinite train of pulses, and the output was presented as an "A" type, or deflection-modulated, trace. The following results were obtained:

Feedback factor: 0.98
Ratio of spurious signal to main signal: -16 db.
Relative distortion: not measured

Referring to Table I, we see that the average buildup of a spurious signal is 16 db at a feedback factor of 0.98. Thus, we would expect a spurious-to-main-signal ratio of -19 db $\pm$ 3 db. The measured ratio of -16 db is within the limit of experimental error.

B. FM Sweep Integrator Using a 667-Microsecond Delay Line

This second FM sweep integrator was constructed under Amendment 3 to Subcontract 16. The delay line was made by Laboratory for Electronics and had 40-mc transducers. By this time, the importance of all the secondaries was appreciated and more accurate measurements were made. It is believed that all measurements made in this experiment were accurate within ± 1 db. The secondary responses of the delay line were measured by M. Axelbank and J. J. G. McCue of Lincoln Laboratory. These are given in Table II.

TABLE II.—*Delay-Line Secondaries Measured at 40 mc, Pulsed Carrier Input.*

Time of Occurrence, μsec	Attenuation, db below main signal	Time of Occurrence, μsec	Attenuation, db below main signal
48	70	376	75
141	80	425	78
143	80 (pair)	473	66 (largest secondary excluding 3rd-round echo)
176	72	525	78
237	74	570	78
331	80	572	76
661	74	860	71
669	0 (main signal)	954	69
673	71	—	68
676	74	—	72
763	74	—	77
765	77	—	78
859	74	—	55 (3rd-round echo)

Sum of secondaries: 45 db below main signal
Square root of sum of squares: 53 db below main signal

It should be noted especially that the arithmetic sum of the secondaries is 45 db below the main signal and the rms sum is 53 db below the main signal.

In the second FM sweep integrator, the following results were obtained:

Feedback factor: 0.98.
Ratio of largest spurious signal to main signal: − 50 db.
Observed relative distortion: − 36 db.

A glance at Table I shows that the expected buildup is 16 db. The largest secondary is − 66 db. The observed result for relative spurious signal was − 50 db, verifying the expected result. For the distortion, the expected relative distortion, according to the analysis in this paper,

was − 53 db + 16 db = − 37 db. The observed ratio was − 36 db, which is within the experimental error.

C. FM Sweep Integrator Using a 500-Microsecond Delay Line

In this delay line, the largest secondary response was 54 db below the main signal on a single pass through the line. At a feedback factor of 0.98, the expected ratio of spurious signal to main signal was:

For random buildup: − 38 db
For coherent buildup: − 20 db

The filaments of all tubes were supplied with direct current and an oscillator was used to introduce a controlled hum deviation on the deviable oscillator. The following results were obtained:

1. DC filaments with no hum deviation
   Ratio of spurious to main signal: − 21.5 db
2. DC filaments with hum deviation
   Ratio of spurious to main signal: − 38 db

Note that there is a small discrepancy of 1.5 db in the coherent-buildup comparison.

VIII. SUMMARY AND CONCLUSIONS

It is unfortunate that I cannot quote quantitative results of experiments with AM sweep integrators. The effects of secondaries in an AM sweep integrator were observed, however, on a PPI presentation; they appear as range rings.

The experiments with FM sweep integrators show that it is justifiable to assume random buildup of secondary responses provided that there exists a small amount of hum deviation of the carrier frequency. The hum frequency should not be an integral multiple of the repetition frequency. If d-c. filaments are used, it may be necessary to introduce a controlled amount of hum. Of course, the possibility exists that the hum will be observed in the output as noise, but it may sometimes be filtered, and in any case it ought to be below the level of fluctuation noise.

Convenient formulas for estimating the dynamic range of sweep integrators, neglecting fluctuation noise, are:

A. AM Sweep Integrator:

$$\text{Dynamic range (db)} = a\text{(db)} + 20 \log_{10}(1 - k) \qquad (14)$$

B. FM Sweep Integrator:

$$\text{Dynamic range (db)} = a\text{(db)} + 10 \log_{10}(1 - k^2) - 2 \qquad (15)$$

where a(db) is the ratio of primary signal to largest secondary expressed in decibels, and k is the feedback factor.

While it is gratifying to obtain verification of the theoretical analysis, I must sound a few words of caution. The phasing of the successive secondary ripples in an FM sweep integrator operating continuously depends on the hum frequency, the hum magnitude, and the time separation between the primary signal and the secondary signal. It is conceivable that this time separation may be so short that the hum magnitude or hum frequency must be made so great as to become noticeable and objectionable in the output.

Sometimes it may be possible to reduce the secondary effects in an FM sweep integrator by restricting the video passband. The distortion and spurious signals contain many frequency components; one of these components is equal to the maximum deviation and this component has the largest amplitude. There are significant contributions from frequency components which are near the maximum deviation frequency. If these frequencies are high enough, some of them may be removed by filtering without significantly affecting the main signal.

APPENDIX

I. Delay-Line Secondary Responses in FM Sweep Integrators and Iterators

If the delay-line input signal is $e_1(t)$ and the output is $e_0(t)$, then:

$$e_0(t) = \sum_i C_i e_1(t - \tau_i) \tag{A-1}$$

$$= C_0 e_1(t - \tau_0) + \sum_{j \neq 0} C_j e_1(t - \tau_j), \tag{A-2}$$

where

C_0 = relative transmission of main path,

τ_0 = time delay of main path,

C_j = relative transmission of j^{th} secondary path, and

τ_j = time delay of j^{th} secondary path.

If the bandwidth of the delay line is large compared with the reciprocal of the pulse duration, the C's are essentially independent of frequency.

The effect of these secondary responses upon integration and cancellation has been considered by Axelbank (**7**). He studied both AM and FM transmission through the delay line and derived the maximum possible signal distortion resulting from secondaries in an FM sweep integrator. Axelbank's result represents a limit; how closely this limit is approached depends upon how the secondaries build up upon recirculation in the integrator loop. For the FM sweep integrator, it is felt that the secondary responses are randomly phased during buildup. Thus, the determination of secondary effects becomes a statistical problem.

Some of Axelbank's formulas will be given here. Let the input to the delay line be a frequency-modulated carrier of constant amplitude as follows:

$$e_1(t) = R \cos \theta(t). \tag{A-3}$$

The delay-line output will be:

$$e_0(t) = RC_0 \cos \theta(t - \tau_0) + \sum_{j \neq 0} RC_j \cos \theta(t - \tau_j). \tag{A-4}$$

In any delay line useful for sweep integrators,

$$\sum_{j\neq 0} C_j \ll C_0. \tag{A-5}$$

Now, let

$$\begin{aligned} \theta(t-\tau_0) &= \theta_0, \\ \theta(t-\tau_j) &= \theta_j. \end{aligned} \tag{A-6}$$

The instantaneous frequency of $e_0(t)$ can be obtained by a simple extension of a formula given by Goldman (8). The instantaneous (angular) frequency is:

$$\psi = \dot{\theta}_0 + \frac{1}{C_0}\sum_{j\neq 0} C_j(\dot{\theta}_j - \dot{\theta}_0)\cos(\theta_j - \theta_0), \tag{A-7}$$

where the dots represent time derivatives. If $\omega(t)$ represents the instantaneous frequency of the delay-line input,

$$\theta(t) = \int_0^t \omega(x)dx. \tag{A-8}$$

Then,

$$\theta_j = \theta(t-\tau_j) = \int_0^{t-\tau_j} \omega(x)dx \tag{A-9}$$

and $\dot{\theta}_j = \omega(t-\tau_j)$.

$$\theta_0 - \theta_j = \int_{t-\tau_j}^{t-\tau_0} \omega(x)dx. \tag{A-10}$$

The frequency of the delayed signal is:

$$\psi = \omega(t-\tau_0) + \frac{1}{C_0}\sum_{j\neq 0} C_j[\omega(t-\tau_j) - \omega(t-\tau_0)]\cos\left[\int_{t-\tau_0}^{t-\tau_j}\omega(x)dx\right]. \tag{A-11}$$

A video signal causes a frequency deviation in the carrier wave. The frequency deviations will be converted to video signals by a frequency discriminator. The video signals will be identified with the corresponding frequency deviations. Let the video input signal to the delay line be $I(t)$, so that:

$$I(t) = \omega(t) - \omega_0 \tag{A-12}$$

where ω_0 is the carrier or undeviated frequency. The resulting frequency deviation of the delay-line output is:

$$\begin{aligned} H(t) &= I(t-\tau_0) \\ &\quad + \frac{1}{C_0}\sum_{j\neq 0} C_j[I(t-\tau_j) - I(t-\tau_0)]\cos\left[\omega_0(\tau_0-\tau_j) + \int_{t-\tau_0}^{t-\tau_j} I(x)dx\right]. \end{aligned} \tag{A-13}$$

The first term on the right-hand side of Eq. A-13 represents the main signal and the terms in the summation are the unwanted secondaries. The nature of these products and their effect upon performance depends upon the input signal. For example, let the input signal be a rectangular pulse of duration δ and let it cause a deviation ω_D. Then,

$$\left.\begin{aligned} I(t) &= \omega_D, && \text{for } 0 < t < \delta \\ &0, && \text{otherwise} \end{aligned}\right\}. \tag{A-14}$$

Assume that δ is small enough so that $I(t-\tau_j)$ and $I(t-\tau_0)$ do not overlap for $j \neq 0$. Under these conditions, the integral in the argument of the cosine in Eq. A-13 has the following value:

$$\left.\begin{aligned} \int_{t-\tau_0}^{t-\tau_j} I(x)dx &= -\omega_D(t-\tau_0), && \text{for } 0 < t-\tau_0 < \delta \\ &= +\omega_D(t-\tau_j), && \text{for } 0 < t-\tau_j < \delta \\ &= 0, && \text{otherwise} \end{aligned}\right\}. \tag{A-15}$$

Then, for $I(t)$ given by Eq. A-14, Eq. A-13 can be written as:

$$H(t) = I(t - \tau_0) - I(t - \tau_0) \sum_{j \neq 0} \frac{C_j}{C_0} \cos [\omega_0(\tau_0 - \tau_j) - \omega_D(t - \tau_0)], \quad \text{for } 0 < t - \tau_0 < \delta \tag{A-16a}$$

$$= \sum_{j \neq 0} \frac{C_j}{C_0} I(t - \tau_j) \cos [\omega_0(\tau_0 - \tau_j) + \omega_D(t - \tau_j)], \quad \text{for } 0 < t - \tau_j < \delta \tag{A-16b}$$

$$= 0, \quad \text{otherwise} \tag{A-16c}$$

In Eq. A-16*a*, the summation on the right-hand side represents the distortion products. The terms of this summation are called distortion products because they all exist at the same time as the signal output and they cause a distortion in the shape of the output pulse. Each distortion term has the form:

$$\frac{C_j}{C_0} \cos (\omega_D t - \varphi_j) \tag{A-17}$$

where φ_j is a phase angle, and thus the distortion appears as a sinusoidal wiggle on the top of the pulse, with a frequency equal to the deviation. The maximum possible amplitude of the distortion would occur when all the terms are in phase; that is, when φ_j is a constant for all j. Then:

$$\text{Maximum relative distortion} = \frac{1}{C_0} \sum_{j \neq 0} C_j. \tag{A-18}$$

The phase, φ_j, is given by:

$$\varphi_j = \omega_D \tau_j + \omega_0(\tau_j - \tau_0). \tag{A-19}$$

The maximum distortion cannot occur unless the various φ_j differ by multiples of 2π. Conceivably, there exists a certain pattern of secondaries which will produce this effect when combined with the correct values of ω_0 and ω_D. It does not seem reasonable that this will occur frequently. It is more plausible to treat the φ_j as random. If they are so treated, then statistical analysis must be applied. This aspect will be treated in greater detail later, but there is one quantity which will be important. This is the so-called relative rms sum, given by

$$\text{Relative rms sum} = \frac{1}{C_0} \left(\sum_{j \neq 0} C_j^2 \right)^{1/2}. \tag{A-20}$$

In Eq. A-16*b*, the terms occur at times which are different from the time of occurrence of the main signal. Also, the individual terms occur at different times, because of the assumption of no overlap for a short input pulse. Each of these "spurious" signals has a relative amplitude C_j/C_0, and each shows a sinusoidal variation equal to the frequency deviation. Then,

$$\text{Maximum relative spurious signal} = C_M/C_0 \tag{A-21}$$

where C_M is the amplitude of the largest spurious signal. Figure 3*b* shows the appearance of a typical frequency-detected output of a delay line with pulsed FM input.

If the input pulse is extended in time, the situation is somewhat different. Let it be assumed that the input pulse duration is great enough so that there is overlap of some of the secondary responses. As the duration of the input pulse is increased, $I(t - \tau_0)$ and $I(t - \tau_j)$ overlap to some extent for one or more values of j. Thus, according to Eq. A-13, the amplitude of distortion would decrease. On the other hand, the spurious signals would increase in amplitude as more secondaries overlap at the same point in time. As the pulse duration becomes very large, the distortion and false signals change roles. Then for an extended pulse,

$$\text{Maximum relative distortion approaches } C_M/C_0 \tag{A-22}$$

$$\text{Maximum relative spurious signal approaches } \frac{1}{C_0} \sum_{j \neq 0} C_j \tag{A-23}$$

$$\text{Relative rms sum of spurious signals approaches } \frac{1}{C_0} \left(\sum_{j \neq 0} C_j^2 \right)^{1/2}. \tag{A-24}$$

II. FM Iteration and FM Integration

When the delay line is used in an FM sweep integrator, successive inputs are added. The secondary responses are also added and are built up to a greater relative extent than the signal. We shall first consider FM *iteration*, which is performed by adding n successive signals with the same weighting factor and then reading out the sum. The device is then wiped clean and the operation repeated. We shall then consider *FM integration*, in which the system operates continuously, each output multiplied by a feedback factor k and then added to the input. Again, we use some of Axelbank's notation (7).

Referring to Eq. A-13, let

$$a_{jn} = \cos\left[\omega_0(\tau_0 - \tau_j) + \int_{t-\tau_0}^{t-\tau_j} I_n(x)dx\right], \tag{A-25}$$

where $I_n(t)$ is the n^{th} input pulse. Then, if $H_n(t)$ is the n^{th} output pulse, Eq. A-13 allows one to write:

$$H_n(t) = I_n(t) + \frac{1}{C_0}\sum_{j\neq 0} C_j a_{jn}[I_n(t - \tau_j) - I_n(t)]. \tag{A-26}$$

At this point, the procedure differs for iteration and integration; we consider iteration first.

A. FM Iteration

In an iterator:

$$I_n(t) = I_1(t) + H_{n-1}(t). \tag{A-27}$$

Therefore,

$$H_n(t) = I_1(t) + H_{n-1}(t) + \frac{1}{C_0}\sum_{j\neq 0} C_j a_{jn}[I_1(t - \tau_j) - I_1(t)] + \frac{1}{C_0}\sum_{j\neq 0} C_j a_{jn}[H_{n-1}(t - \tau_j) - H_{n-1}(t)]. \tag{A-28}$$

Neglecting terms in C_jC_k/C_0^2,

$$H_n(t) = nI_1(t) + \frac{1}{C_0}\sum_{j\neq 0} C_j\left(\sum_{r=1}^{n} ra_{jr}\right)[I_1(t - \tau_j) - I_1(t)]. \tag{A-29}$$

For a pulse of short duration, there is no overlap among the various terms of Eq. A-29. It can then be written as follows:

$$H_n(t) = nI_1(t) - \frac{I_1(t)}{C_0}\sum_{j\neq 0} C_j\left(\sum_{r=1}^{n} ra_{jr}\right) + \frac{1}{C_0}\sum_{j\neq 0} C_j\left(\sum_{r=1}^{n} ra_{jr}\right) I_1(t - \tau_j). \tag{A-30}$$

The first summation of the right-hand side of Eq. A-30 represents the built-up distortion products, all of which occur at the same time as the main signal. The terms of the second summation are the individual built-up spurious signals, having no overlap in time. We shall concentrate on the largest of these.

The maximum values of distortion and spurious signals are obtained when the a_j's are independent of r; that is, when all the a_j's have the same phase. (See Eq. A-30.) Now, is it valid to assume that the a_j's all have the same phase? The answer is no. Common phasing ought to occur very rarely. Let us write Eq. A-25 in a slightly different form. Assuming that the iterator input pulses are rectangular with duration δ, and the deviation of each pulse is ω_D, Eq. A-25 becomes:

$$\left.\begin{aligned} a_{jr} &= \cos[\omega_0(\tau_0 - \tau_j) + r\omega_D\tau_0 - r\omega_D t], \quad \text{for } 0 < t - \tau_0 < \delta \\ &= \cos[\omega_0(\tau_0 - \tau_j) + r\omega_D t - r\omega_D\tau_j], \quad \text{for } 0 < t - \tau_j < \delta \end{aligned}\right\} \tag{A-31}$$

For successive values of r, the phase angle, $r\omega_D\tau_0$ or $r\omega_D\tau_j$, changes. A typical value of ω_D may be near 20 kc. Now τ_0 or τ_j may be many microseconds. For τ_0 or τ_j equal to 100 microseconds,

$$\omega_D\tau_j = 4\pi \text{ radians} \tag{A-32}$$

showing that phase changes may be very large.

However, we can see that at $t = \tau_0$ or at $t = \tau_j$, a_{jr} is independent of r and the magnitude of the buildup depends upon the phase angle $\omega_0(\tau_0 - \tau_j)$. If the carrier frequency remains constant, the buildup will be coherent, but only at the initial instant, represented by $t = \tau_0$ or $t = \tau_j$. The various a_j terms quickly get out of phase, so that it is plausible to hypothesize that these are randomly phased except at the initial instant. Now, the carrier frequency may have a small natural instability, representing the internal "hum." Even if this natural hum does not exist, a small amount may be deliberately introduced, as will be discussed in connection with FM sweep integration. In addition, even if there is a tendency for initial coherent buildup to produce an impulse like effect at $t = \tau_0$ or $t = \tau_j$, the inevitable bandwidth limitation will not support such a buildup. Therefore, we make the hypothesis that for all t, the a_j terms are randomly phased and are uniformly distributed in the interval $(0,2\pi)$.

The buildup is thus random. In the case of the individual spurious signals, we may concentrate on the largest one and treat it separately, since there is no overlap of the various secondaries in any one pass through the delay line. In the case of the distortion, however, all the secondaries contribute. It is this total contribution which is built up upon successive recirculations through the delay line. This total contribution is given by the second term on the right-hand side of Eq. A-30 for any one value of r.

On any one pass, distortion terms are:

$$\frac{1}{C_0}\sum_{j\neq 0} C_j \cos\left[\omega_0(\tau_0 - \tau_j) - r\omega_D(t - \tau_0)\right]. \tag{A-33}$$

The phases of the various terms differ because of the different values of j. Again, these phase differences are likely to be very large, and we make the hypothesis that the various terms in Eq. A-33 have random phases uniformly distributed in the interval $(0, 2\pi)$.

Determining the effects of secondaries has become a statistical problem, and we must find the probability density function for the secondaries. If there is a large number of such terms, no one of which has a large contribution compared to the total, the distribution of amplitude is approximately a Rayleigh distribution, with the added restriction that the maximum value cannot exceed the arithmetic sum of the amplitudes of the individual components. Then, the density function is given by:

$$\left.\begin{aligned} W(R) &= \frac{R}{R_0^2}\exp\left(-\frac{R^2}{2R_0^2}\right) + A\delta(R - R_{\max}), \qquad 0 \leq R \leq R_{\max} \\ &= 0, \qquad R > R_{\max}, \qquad R < 0 \end{aligned}\right\} \tag{A-34}$$

where R is the resultant amplitude, R_0 is the rms sum and

$$A = \frac{1}{R_0^2}\int_{R_{\max}}^{\infty} R\exp\left(-\frac{R^2}{2R_0^2}\right)dR = \exp\left(-\frac{R_{\max}^2}{2R_0^2}\right). \tag{A-35}$$

It should be emphasized that Eq. A-34 represents an approximate distribution, which becomes increasingly valid as the number of components increases.

If we let the amplitude of $I_1(t)$ in Eq. A-30 be represented by I_1, we have the following results:

For the largest spurious signal,

$$R_0^2 = \frac{I_1^2 C_M^2}{C_0^2}\sum_{r=1}^{n} r^2 = \frac{I_1^2 C_M^2 n(n+1)(2n+1)}{6C_0^2} \tag{A-36}$$

where C_M/C_0 = relative amplitude of largest spurious response and

$$R_{\max} = \frac{I_1 C_M}{C_0} \sum_{r=1}^{n} r \tag{A-37}$$

$$= \frac{n(n+1)I_1 C_M}{2C_0}.$$

For the resultant distortion,

$$R_0^2 = \frac{n(n+1)(2n+1)I_1^2}{6C_0^2} \sum_{j \neq 0} C_j^2 \tag{A-38}$$

$$R_{\max} = \frac{n(n+1)I_1}{2C_0} \sum_{j \neq 0} C_j. \tag{A-39}$$

Now there arises the question of what is actually observed. Let us imagine that complete traces, each representing n successive additions, are fed to an oscilloscope. If enough traces per second are presented, a steady picture will result. It seems plausible to state that the distortion and spurious signal which will be observed will be the average amplitude in each case. In the case of the Rayleigh distribution, the average value is given by:

$$\bar{R} = \int_0^\infty RW(R)dR = \sqrt{\pi/2}\, R_0 \tag{A-40}$$

or 2 db greater than R_0.

To determine the relative levels of distortion and spurious signal, we must divide by the value of the built-up signal, which is nI_1. We have, then, the following results for the FM iterator:

$$\text{Average relative distortion} = \frac{\sqrt{\pi/2}}{C_0} \sqrt{\frac{(n+1)(2n+1)}{6n} \sum_{j \neq 0} C_j^2} \tag{A-41}$$

$$\text{Average relative maximum spurious signal} = \frac{\sqrt{\pi/2}\, C_M}{C_0} \sqrt{\frac{(n+1)(2n+1)}{6n}}. \tag{A-42}$$

When n is large, the following approximations hold:

$$\text{Average relative distortion} \approx \frac{\sqrt{\pi/2}}{C_0} \sqrt{\frac{n}{3} \sum_{j \neq 0} C_j^2} \tag{A-43}$$

$$\text{Average relative maximum spurious signal} \approx \frac{\sqrt{\pi/2}\, C_M}{C_0} \sqrt{\frac{n}{3}}. \tag{A-44}$$

B. FM Sweep Integration

Turning back to Eq. A-26, we may proceed further by noting that, in an integrator,

$$I_n(t) = I_1(t) + kH_{n-1}(t), \tag{A-45}$$

where k is the feedback factor. Then, substituting Eq. A-45 into Eq. A-26,

$$H_n(t) = I_1(t) + kH_{n-1}(t) + \sum_{j \neq 0} \frac{C_j a_{jn}}{C_0} [I_1(t - \tau_j) - I_1(t) + kH_{n-1}(t - \tau_j) - kH_{n-1}(t)]. \tag{A-46}$$

Neglecting terms in $C_j C_1 / C_0^2$ leads to:

$$H_n(t) = \frac{1 - k^n}{1 - k} I_1(t) + \sum_{j \neq 0} \frac{C_j}{C_0} [I_1(t - \tau_j) - I_1(t)] \sum_{i=1}^{n} \frac{a_{ji} k^{n-i}(1 - k^i)}{1 - k}. \tag{A-47}$$

In order to get specific results, we consider what happens when $n \to \infty$. Then:

$$H_\infty(t) = \frac{I_1(t)}{1-k} - \sum_{j\neq 0} \frac{C_j}{C_0} I_1(t) \lim_{n\to\infty} \sum_{i=1}^{n} \frac{a_{ji}k^{n-i}(1-k^i)}{1-k}$$

$$+ \sum_{j\neq 0} \frac{C_j}{C_0} I_1(t-\tau_j) \lim_{n\to\infty} \sum_{i=1}^{n} \frac{a_{ji}k^{n-1}(1-k^i)}{1-k}. \quad \text{(A-48)}$$

The first summation on the right-hand side of Eq. A-48 represents the distortion and the second summation represents the spurious signals. For the short pulse which we have assumed, there is no overlap. Now, consider the summation:

$$\frac{1}{1-k} \sum_{i=1}^{n} a_{ji}k^{n-i}(1-k^i). \quad \text{(A-49)}$$

From Eq. A-25,

$$a_{jn} = \cos\left[\omega_0(\tau_0 - \tau_j) + \int_{t-\tau_0}^{t-\tau_j} I_n(x)dx\right].$$

For a rectangular pulse,

$$I_n = \omega_{Dn} \quad \text{(A-50)}$$

which represents the total deviation after n pulses have entered the system. Then

$$\begin{aligned} a_{jn} &= \omega_0(\tau_0 - \tau_j) - \omega_{Dn}(t-\tau_0), \qquad 0 < t-\tau_0 < \delta \\ &= \omega_0(\tau_0 - \tau_j) + \omega_{Dn}(t-\tau_j), \qquad 0 < t-\tau_j < \delta \\ &= \omega_0(\tau_0 - \tau_j), \qquad \text{otherwise.} \end{aligned} \quad \text{(A-51)}$$

As n grows very large, the value of the deviation approaches an asymptote, given by $\frac{\omega_D}{1-k}$. The successive values of ω_{Dn} become indistinguishable from one another so that we may write $\omega_{Dn} = \omega_{D(n-1)} = \omega_{D(n-2)} = \cdots$. Then all of the a_j in Eq. A-49 are the same, and the summation becomes, in the limit as $n \to \infty$:

$$\frac{1}{(1-k)^2} a_{jn}. \quad \text{(A-52)}$$

This is a case of coherent buildup; the assumption of random phasing for the different a_j is not justified as long as a_{jn} is independent of n for large n. Examination of Eq. A-51 shows that as long as ω_0, the carrier frequency, is independent of n, a_{jn} is independent of n and there is coherent buildup. However, if the carrier frequency is not constant, the various a_j might be different. The natural hum level on the deviable oscillator may be enough to ensure random phasing of the various a_j. With this in mind, Eq. A-51 can be expressed, for the time interval $0 < t - \tau_j < \delta$, as:

$$a_{jn} = \omega_{0n}(\tau_0 - \tau_j) + \omega_{Dn}(t - \tau_j) \quad \text{(A-53)}$$

where ω_{0n} is the carrier frequency for the n^{th} input pulse. For each 100 microseconds difference between τ_0 and τ_j, a carrier frequency change of 10 kc changes the successive phases by 2π radians.

The natural hum level may be insufficient to ensure random buildup, particularly if d.c. is used for the vacuum-tube filaments. Hum can then be introduced deliberately in just the amount necessary to obtain random buildup.

Following the arguments given before, we state that the a_{ji} terms have phases uniformly distributed in the interval $(0, 2\pi)$. Thus, the sum in Eq. A-49 is Rayleigh distributed for large n. For the resultant distortion, the parameter R_0^2 is given by:

$$R_0^2 = \frac{1}{(1-k)^2} \sum_{i=1}^{n} (k^{n-i} - k^n)^2 \left(\frac{I_1^2}{C_0^2} \sum_{j\neq 0} C_j^2\right). \quad \text{(A-54)}$$

For $n \to \infty$, this reduces to:

$$R_0^2 = \left(\frac{I_1^2}{C_0^2} \sum_{i \neq 0} C_i^2 \right) \frac{1}{(1-k)^2(1-k^2)}. \tag{A-55}$$

For the largest spurious signal,

$$R_0^2 = \frac{C_M^2 I^2}{C_0^2(1-k)^2(1-k^2)}. \tag{A-56}$$

The amplitude of the signal after buildup is:

$$\frac{I_1}{1-k} = S_M. \tag{A-57}$$

Then the average relative value of distortion or spurious signal is given by:

$$\frac{\sqrt{\pi/2}\, R_0}{S_M}. \tag{A-58}$$

A sweep integrator operates continuously. If we imagine that the successive sweeps are fed to an oscilloscope with an A-type presentation, a steady picture will result if there are enough sweeps per second. As before, it seems plausible to state that the distortion and spurious signal which will be observed will be the average amplitude.

Then we have the following results for the FM integrator:

$$\text{Average relative distortion} = \frac{\sqrt{\pi/2}}{C_0} \sqrt{\frac{\sum_{i \neq 0} C_i^2}{1-k^2}} \tag{A-59}$$

$$\text{Average relative maximum false signal} = \frac{\sqrt{\pi/2}\, C_M}{C_0 \sqrt{1-k^2}}. \tag{A-60}$$

In either case the average relative buildup caused by integration is given by the factor:

$$\sqrt{\frac{\pi}{2(1-k^2)}}. \tag{A-61}$$

This is tabulated in Table I for various values of k. Also given in Table I is the maximum possible value of relative buildup, given by:

$$\lim_{n \to \infty} \frac{1}{1-k} \sum_{i=1}^{n} k^{n-i}(1-k^i) = \frac{1}{1-k} \tag{A-62}$$

Such a buildup would occur if all the a_j terms had identical phase. This can be called "coherent" buildup.

REFERENCES

(1) L. N. RIDENOUR, "Radar System Engineering," Vol. 1, MIT Radiation Laboratory Series, New York, McGraw-Hill Book Co., Inc., 1947, pp. 38–47.

(2) T. T. EATON, "Noise Reduction by Delayed Feedback," Final Report, OSRD Contract OEM-or-252, RCA Laboratories, Princeton, N. J., April 15, 1943.

(3) J. V. HARRINGTON AND T. F. ROGERS, "Signal-to-Noise Improvement Through Integration in a Storage Tube," *Proc. IRE*, Vol. 38, pp. 1197–1203 (1950).

(4) H. URKOWITZ, "Video Integration of Pulsed Radar Signals," M.S. Thesis, Moore School of Electrical Engineering, University of Pennsylvania, February, 1953; also published as Philco Corporation Research Report No. 210M, February, 1953.

(5) D. E. SUNSTEIN, W. USSLER AND A. C. MUNSTER, "Signal Integrating System," U. S. Patent No. 2,841,704, July 1, 1958.

(6) D. E. SUNSTEIN, "Signal Integrating System," U. S. Patent No. 2,736,021, February 21, 1956.

(7) M. AXELBANK, "Delay-Line Secondary Responses and Their Effect on Cancellation and Integration," Group Report 23.7, MIT Lincoln Laboratory, April, 1954.

(8) S. GOLDMAN, "Frequency Analysis, Modulation, and Noise," New York, McGraw-Hill Book Co., Inc., 1948, p. 165.

A SERVOMECHANISMS APPROACH TO SKILL DEVELOPMENT *

BY

EZRA S. KRENDEL [1] and DUANE T. McRUER [2]

SUMMARY

Motor skills development is explained in terms of perceptual organization. As skills develop, Successive Organizations of Perception (SOP) enable the operator to take increasing advantage of the redundancy or predictability of his input signals. A servomechanism model is presented to describe how the predictability of input signals could be made more apparent to the human operator by the use of appropriate displays. The hypothesis is advanced that this external organization of input signals achieved by displays is a model for the effective SOP achieved as skill is acquired. Experimental evidence to support this SOP model is presented from the literature on manual tracking. Implications for future research are discussed.

I. INTRODUCTION

Skill is a composite measure of human proficiency in terms of task criteria (such as precision and speed of response and repeatability) and man-centered criteria (such as economy of effort). Both perceptual and motor performance are involved, and the relative importance of these two aspects has been discussed at considerable length in the literature. This paper is oriented toward a perceptual approach to skills in the sense recently outlined by Annett and Kay (**1, 2**).[3] The perceptual process is defined broadly so as to receive stimuli from these sources: a neutral external environment, a human operator, and an associated external environment. Stimuli from the neutral external environment appear as unalterable signals on an external display which also includes signals resulting from operator manipulation of the associated external environment. Response-associated stimuli from the musculature and nervous system are also present in this definition of perception, although these are not indicated on the external display.

The purpose of this paper is to elaborate a model for the development of eye–hand skills in the context of servomechanism theory which is consistent with both the perceptual approach to skills and such data as are available from manual control studies. This model was first presented in an abbreviated and limited form in (**3**).

* This research was supported in whole or in part by the United States Air Force under Contracts AF 33(616)-5408 and AF 33(616)-5822, monitored by Aero Medical Laboratory, Directorate of Research, Wright Air Development Center, Wright Patterson Air Force Base, Ohio.

[1] Head, Engineering Psychology Branch, The Franklin Institute Laboratories for Research and Development, Philadelphia, Pa.

[2] Technical Director, Systems Technology, Inc., Inglewood, Calif.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

To make use of a fruitful source of data on skilled human performance in visual-motor coordination this discussion will be limited to that type of task called manual tracking—the manual process of minimizing visually perceived errors by exercising continuous control so as to match visually presented input and output signals. Since, in the tracking context, the associated external environment is the controlled element and the neutral external environment is the system forcing function, these stimuli sources can be rigorously controlled experimental variables. The tracking data which we discuss in the following sections are those for which the associated external environment, that is, controlled element, is fixed as a pure gain. By limiting the variables our exposition of the critical influence of the neutral external environment on the SOP model for skill development can proceed more smoothly.

On an intuitive basis the model can be extended into situations with other controlled elements and forcing functions, although experimental data to support or deny this extension do not yet exist. Some attention is, therefore, devoted to mention of additional experiments which would be desirable to fill out and/or more quantitatively limit the application of the model.

II. STIMULUS ORGANIZATION

A. Displays for Organizing Stimuli

In a perceptual approach to skill development the display selected is critical. By appropriate display design, redundancies in the signals from the external environment may be made more readily apparent to the observer. In a tracking context the characteristics of the display provide a basis for the classification of tracking problems. Our classification criteria will be based on the type of information which the stimulus to be tracked and the controller's response present to the controller as a basis for future tracking decisions. These criteria result in three limiting types of classifications, defined as follows:

1. Compensatory

In compensatory tracking the visually displayed effects of the controller's responses are not distinguishable from the system's input. The visual display is the system forcing function minus the modified control response, so the operator can determine the effects of control motion alone only under zero input conditions.

2. Pursuit

In pursuit behavior past experience provides the tracker with information about what to expect in a future input, but he must operate in a closed loop fashion with visual feedback about his responses. In

this form of tracking the operator's corrective responses can be distinguished from his input.

3. Precognitive

This condition exists when the operator has effectively complete information about the input's future. A stimulus can then serve to trigger off a repertory of practiced, properly sequenced responses on the part of the operator. Continuous close control is not maintained on the perceived error, so precognitive behavior is not tracking. However, we have included this term because occasionally tracking approaches these conditions. Precognitive behavior is essentially open loop control.

These three types of tracking correspond to three different displays or organizations of the external stimuli. In Fig. 1, displays and functional block diagrams are presented for compensatory tracking and for pursuit tracking. For an example of a precognitive display consider the driver's view of a road whose curves, dips and rises are all visible far ahead so that appropriate maneuvers can be begun in advance. Such a display would enable the operator to function in an open loop fashion for periods of time which would depend on the speed of the automobile, the dynamics of the automobile, and the path the road takes. A pure form of a precognitive display is an idealization. Pure compensatory or pursuit systems as shown in Fig. 1 are also limiting examples. In the compensatory display the operator is presented with an input consisting only of an indicator showing the difference, or error, $\epsilon(t)$,† between the forcing function $i(t)$ and the system output, $r(t)$. The operator's task is to minimize the error signal presented by trying to keep the circle superimposed on the stationary dot. In the pursuit display the operator sees both the input and output of the system. Again the operator's task is to minimize the error existing between the location of the dot and the circle, with the general operation being one of pursuing the target with the follower, trying to keep the circle around the dot. For the sake of completeness we have included controlled elements in Fig. 1, but we will actually concern ourselves in this paper with conditions for which $Y_c = 1$. The controlled element includes the type of vehicle or mechanism under control and the actual means for exerting control.

B. A Successive Organization of Perception Model for Learning

It is the thesis of this paper that eye–hand coordination skill development may be idealized as a progression from a compensatory display organization through a pursuit type organization culminating in an open loop response, as the result of a stimulus organization internal to the operator. Thus,

† A list of symbols used is appended to this paper.

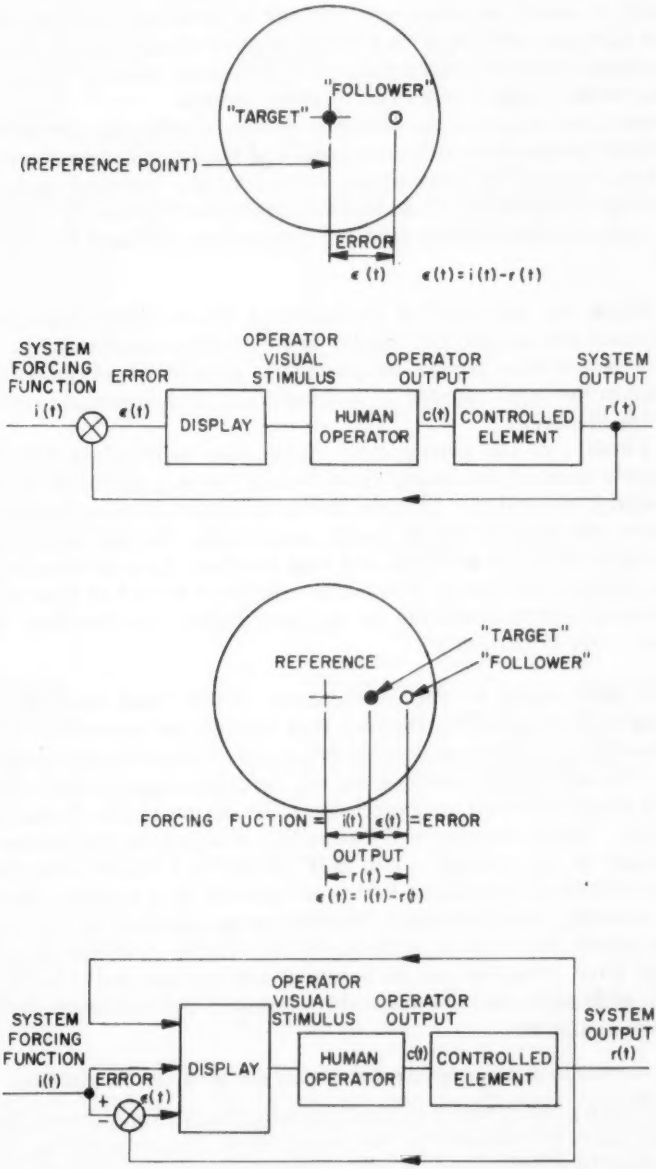


FIG. 1. Compensatory (at top) and pursuit closed loop manual tracking systems: schematic displays and functional block diagrams.

although it would be extremely difficult to achieve, it is conceivable that an operator working with a compensatory display could ultimately demonstrate "effective" open loop control in some tasks by dint of much training with a limited repertory of input stimuli.

Perfect prediction of the stimulus makes possible the pre-setting in the neural centers of an entire response, and the substitution of monitory open loop control for continuous control by the external senses. In the process of setting up these learned responses, whether in tracking or other tasks, we hypothesize that the human goes through the following stages:

1. When the input is first encountered, all control is dependent on the external senses, and the display is effectively compensatory.
2. At some later period, an effectively pursuit situation evolves as the human becomes capable of making short time interval predictions about his display.
3. Finally, in the precognitive mode, the appropriate response is completely learned, becoming synchronous (with a perfectly predicted input signal) or skilled. In some instances observed experimentally an apparent precognitive mode could conceivably be the same type of control as 2, with the addition of a lead to offset the neuromuscular lag. A true precognitive mode, which can exist for a period of time without the external senses operating in the loop, cannot be described in this fashion. (By definition!)

This SOP model for the development of eye–hand coordination or tracking skills emphasizes the fact that the human operator's function in a closed loop tracking task *is most significantly that of a data-organizing device.* In subsequent sections we will describe experimental effects in manual tracking which are functions of the nature of the display or the stimulus. These changes are essentially changes in the external organization of the stimuli. The SOP model is a hypothesis that the human's internal organizing functions operate in a manner similar to these external, experimentally feasible, manipulations of the display. To the extent that system behavior is compatible with this hypothesis, we will have achieved an SOP model for human skill development which, although not demonstrably unique, is consistent with the evidence available.

III. PURSUIT AND COMPENSATORY DISPLAYS FOR STIMULUS ORGANIZATION

To make predictions of manual tracking results for different displays it is necessary to convert the functional block diagrams of Fig. 1 to the equivalent block diagrams of Fig. 2 which show a more detailed operation of the system. The rationale for this quasi-linear description of the human operator is presented in (3 or 4). To simplify the structure in

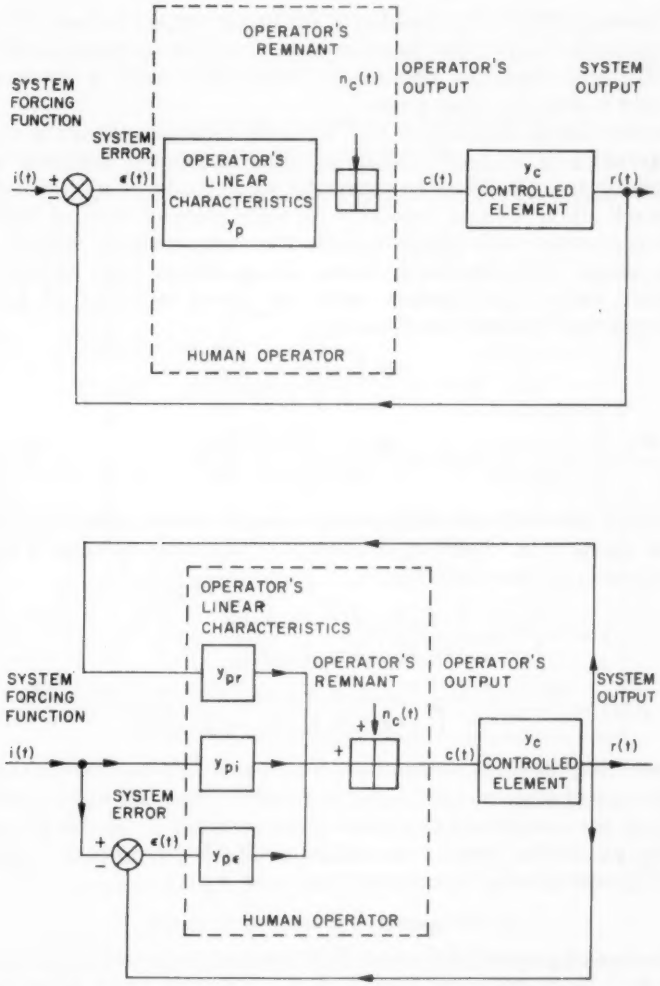


FIG. 2. Equivalent block diagrams with linearized operators for compensatory (at top) and pursuit systems.

Fig. 2, the dynamic characteristics of the display are lumped with the controlled element, and the actual forcing function is modified, if necessary, to an equivalent one. The linear transfer characteristics of the human operating on his presented inputs are described by the weighting functions, $y_{pi}(\tau)$, $y_{p\epsilon}(\tau)$, $y_{pr}(\tau)$, and $y_p(\tau)$, or their Laplace transforms $Y_{pi}(s)$, $Y_{p\epsilon}(s)$, $Y_{pr}(s)$ and $Y_p(s)$. Since all of the operator's output is not described by the action of these transfer characteristics,

an additional term, $n_c(t)$, is added at his linear output to form the total operator output $c(t)$. The location of $n_c(t)$ at the operator's output is arbitrary, and does not necessarily imply that such a quantity is physically inserted at that point.

The two block diagrams of Fig. 2 clearly illustrate the servo system characteristics of manual control systems in general tracking tasks. Recognizing this allows us to apply the whole body of servo theory in our attack upon human behavior in such manual control systems. As servo systems with single inputs, the compensatory system is of simple, single unity-feedback form. As a closed loop system, the operator's output and system error are given in terms of Laplace transforms and transfer functions by,

$$\begin{aligned} C(s) &= \frac{Y_p(s)I(s) + N_c(s)}{1 + Y_p(s)Y_c(s)}, \\ E(s) &= \frac{I(s) - Y_c(s)N_c(s)}{1 + Y_p(s)Y_c(s)}. \end{aligned} \tag{1}$$

The pursuit system is a somewhat more complex servo, being a particular type of open-cycle, closed-cycle system. Here the operator's output and system error are given by

$$\begin{aligned} C(s) &= \frac{[Y_{pe}(s) + Y_{pi}(s)]I(s) + N_c(s)}{1 + Y_c(s)[Y_{pe}(s) - Y_{pr}(s)]} \\ E(s) &= \frac{\{1 - Y_c(s)[Y_{pr}(s) + Y_{pi}(s)]\}I(s) - Y_c(s)N_c(s).}{1 + Y_c(s)[Y_{pe}(s) - Y_{pr}(s)]} \end{aligned} \tag{2}$$

By comparing the error equations of Eqs. 1 and 2, it is apparent that the pursuit type of system could result in smaller errors and higher performance than the compensatory system if the operator is capable of making effective use of the greater versatility available by properly adjusting the additional transfer functions $Y_{pr}(s)$ and $Y_{pi}(s)$.

IV. THE LEARNING PROGRESSION IN DETAIL

The learning model of Section II *B* can now be presented in terms of "effective" block diagrams Figs. 3, 4, 5. Effective block diagrams are descriptive geometric constructions which summarize the measured phenomenon. These block diagrams should not be confused with physical entities.

When an operator begins to track an input signal which has internal coherence—that is, predictable or redundant—his "recognition" of this predictability is not immediate. Since we are limiting our discussion to controlled elements of unity dynamics, the habituation period will be short and the operator's training will be directed toward detecting and acting on the coherence in the input signals.

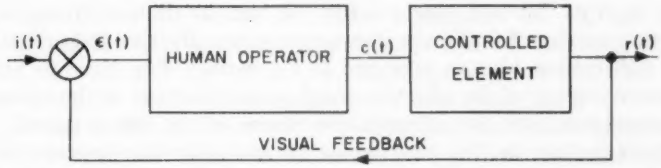


FIG. 3. Compensatory situation representing the initial phase of the progression sequence.

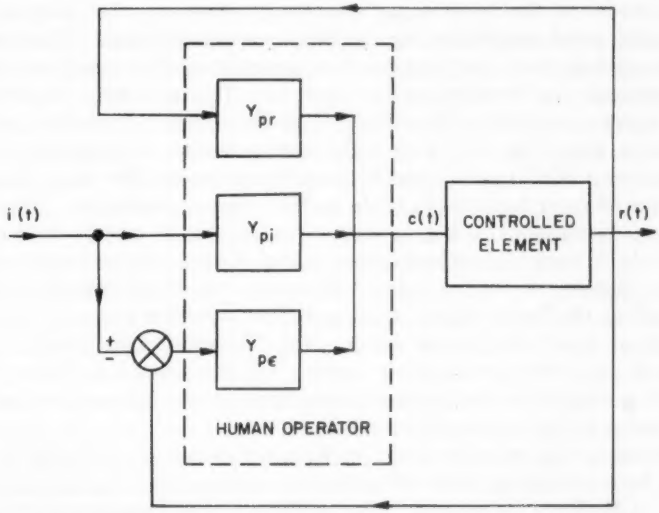


FIG. 4. Pursuit situation representing the second phase of the progression sequence.

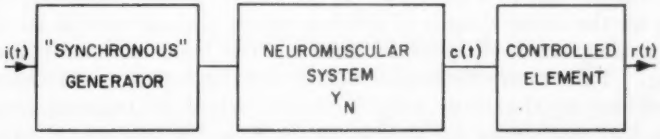


FIG. 5. Open outer loop representing the synchronous phase of the progression sequence.

Consider an operator tracking a compensatory display for which the input has a high degree of coherence. As the operator begins to track, the display is both *actually* and *effectively* compensatory (Fig. 3). This is particularly true at the initial instant before the operator has started his response. If the display were pursuit, the operator could begin rapidly to take advantage of both the nature of the display and internal coherence in the input signal, and would generate a suitable transfer function Y_{pi} to overcome the visual reaction time (Fig. 4). Since the display considered is compensatory, however, the operator must work

a great deal harder and longer before he can, by dint of "recognizing" coherence, achieve by *effective* restructuring an effective pursuit display which will enable him to generate a Y_{pi} term. The time to achieve this restructuring of the effective display is a function of the operator's past experience and the internal coherence of the input signal. The culminating stage in this progression occurs when the operator is able to detect and take advantage of all of the internal coherence and predictability of his input signal (the error). In this case, depending on the extent of the input signal's internal coherence, the operator can accomplish total prediction for varying intervals of time. The central nervous system may be thought of as generating this predicted signal to command the neuromuscular system. The resulting response is based upon a complete "knowledge" of the forcing function and the operator's lags, Fig. 5. This culminating mode of response in the hypothesized SOP model of skill learning is essentially open loop (in the outer loop, at least) with little and infrequent feedback. Thus the operator demonstrates his capacity, based on *a priori* knowledge, effectively to enrich his stimulus input and to diminish his requirements for the display of the complete stimulus. As the predicted motion drifts off or the input signal is changed, the operator retraces his steps through at least the pursuit mode, and "corrects" his "predictor" to get back into the precognitive mode. If the physical display were pursuit, a similar form of internal structuring could also occur causing a progression to the precognitive mode.

The foregoing hypothesized development can be given intuitive substance by considering, in an introspective manner, how certain common motor skills have developed. As an illustration, consider learning to steer an automobile. Many beginners start by relating a fixed object on the automobile to a guideline on the road. Thus a novice, concentrating on the many details of driving, steers the automobile so that a constant distance is maintained between the hood ornament and the curbing. This is an effective *compensatory* display since the operator is attending to the error only. As the driver in training acquires facility and confidence he is able to increase his perceptual capacity for external stimuli, so his effective field of view or perceptual aperture increases. In the course of this process he becomes aware of the separate characteristics of the hood ornament and the curbing. In other words, he can attend to input signal and control response independently. He thus achieves a *pursuit* display type of organization of his visual field, and can make use of the regularities and predictable features of the road. Finally, on achieving complete familiarity with the controlled element, the driver can view the entire visible field, sampling it as necessary, and can steer the automobile as necessary with deft, discrete movements. The interval between these short duration open loop control responses is dependent on the degree to which the

road winds, the speed of the car, and the amount of traffic. Other examples of driving in an open loop fashion for brief intervals of time occur during such emergency maneuvers as steering out of a skid.

The foregoing intuitive example has been confounded to a large extent by the fact that the subject was learning the vehicle's dynamics as well as the regularities inherent in the path he was trying to follow. This confounding, however, causes a process to persist which might not have been as apparent had the dynamics been easy to learn, that is, $Y_c = 1$.

Something skin to the precognitive mode for perfectly predictable stimuli appears in other situations where the operator has thoroughly "learned" his inputs and, by experience, seems to provide exactly the required output. Examples of this "total prediction" are a helmsman's ability in "meeting" a turning ship and some pilots' ability to cope with the well known "JC maneuver," by applying a single, properly timed, conditioned output response. Other skilled activities, like catching a ball and tagging a base runner in one continuous motion, counter-punching, serving a tennis ball, etc., are similar examples.

The training time needed to achieve precognitive or short duration open loop behavior will vary with the task. Since the training time to achieve high proficiency open loop performance will be longer than the time needed to achieve intermediate stages of performance, comparisons between open loop and continuous control tasks are not valid in the foregoing context if equal training time has been allowed for both types of tasks. For example, a child learning to write his name will do better with closed loop control and reference to a standard guide, be it visual or conceptual, at the beginning of his training period. After considerable practice, however, executing his signature will become a simple open loop response.

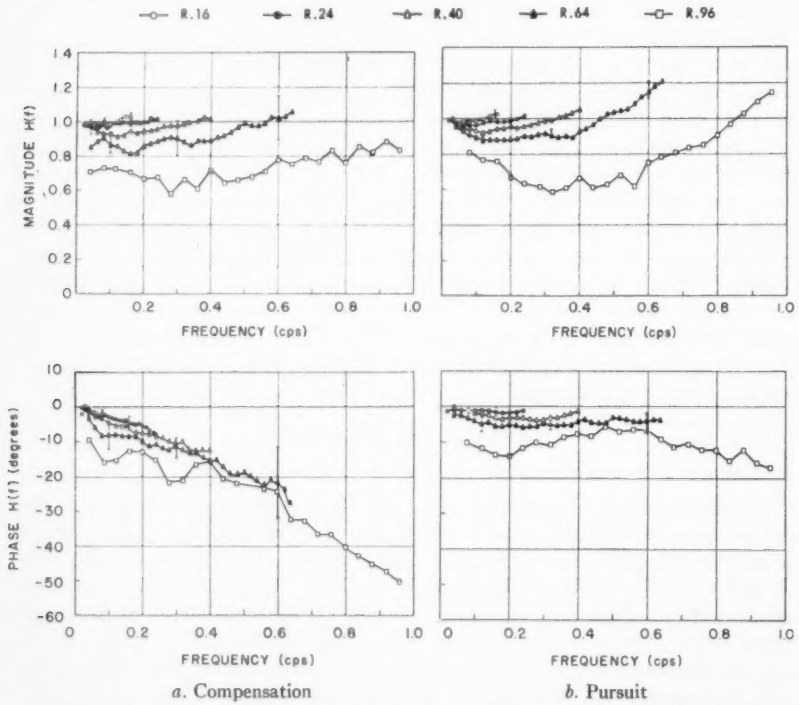
V. EXPERIMENTAL CONFIRMATION

A. Pursuit and Compensatory Stimulus Organization

The possibilities implicit in Eq. 2 have been recognized on a more intuitive level by many investigators and studied without the benefit of human dynamics measurements. Thus Poulton (**5**) demonstrated that in the tracking of a compensatory display, error could be reduced considerably by adding a pointer driven by $i(t)$, the forcing function input. The addition of a pointer driven by $c(t)$, the operator's output, had no significant effect. Sender and Cruzen (**6**) conducted an ingenious experiment to compare pursuit and compensatory backing as well as contrived combinations of the two. Pursuit tracking was always characterized by a lower error and this effect was explained by the fact that the display provided more information on which the operator might base predictive actions. Chernikoff *et al* (**7, 8**) corroborated these findings on comparison between compensatory and pursuit displays

and Chernikoff and Taylor (9) indicated that for very simple tracking tasks, where the added information inherent in a pursuit display was unnecessary, the difference between these displays favored compensatory displays.

The implications in Eq. 2 for changes in the human dynamics are confirmed by Elkind's (10) elegant experimental results. A general and well-known result of human dynamics studies (3 or 4) is that for random appearing inputs and compensatory displays the human operator's open loop describing function contains the factor $e^{-\tau s}$, representing reaction-time lag. This reaction-time delay has important consequences in the closed loop transfer function. Indeed, from the standpoint of form alone, the closed loop transfer function $[H(s) = Y_p/(1 + Y_p)]$ zeros are the same as those of the open loop, so we would expect an $e^{-\tau s}$ term in $H(s)$.



a. Compensation *b*. Pursuit

FIG. 6. Comparison of closed-loop describing functions for Elkind's bandwidth experiment (10). The forcing function spectra used to generate the data in this figure were made up of a large (40 to 80) number of equal amplitude sinusoids of arbitrary phase and equally spaced frequency increments, thereby approximating perfect rectangular spectra. These forcing functions are designated R.16, R.24, etc., where the numerical values refer to the value of f_∞, the highest frequency sinusoid present.

The blocks Y_{pc}, Y_{pi} and Y_{pr} in Fig. 2, unlike Y_p, are not susceptible to evaluation by direct measurement. In lieu of suitable indirect and difficult to obtain measures of these describing functions we can only note their composite effects upon the closed loop results. These effects can be easily seen from Fig. 6 by noting that the pursuit and compensatory phase lags exhibit totally different trends. In particular, as noted above, the closed loop compensatory results reflect strongly the $e^{-\tau s}$ reaction-time delay effect, while the pursuit phases are nearly constant with frequency and are much closer to zero degrees in magnitude. Elkind demonstrated that Y_p (see Fig. 2) as measured by the same forcing functions as were used in the data in Fig. 6 was of the form[4]

$$Y_p = \frac{Ke^{-\tau s}}{(T_1 s + 1)(T_2 s + 1)}.$$

He then showed, assuming that only Y_{pi} and Y_{pc} of Fig. 2 are active, that the decreased phase lag in Fig. 6*b* can be explained quite well by associating a simple first order lead term with the describing function Y_{pi}, while leaving Y_{pc} of the same form as Eq. 3.[5]

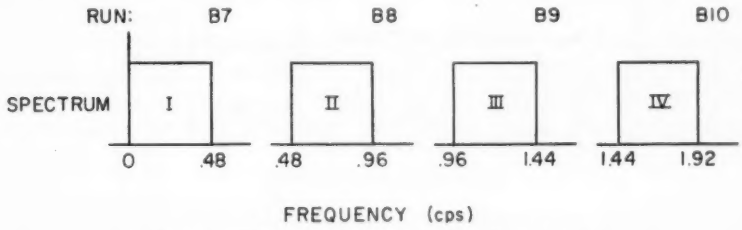


FIG. 7. Elkind's bandpass forcing function spectra.

For our purposes here the important point to be observed is that the possibility for higher performance held forth by the form of Eq. 2 is indeed accomplished in practice by the human's action in utilizing the additional information presented by the display. The major demonstrable effect of the stimulus organization made possible by the pursuit display in this case is the reduction of reaction-time effects upon the closed loop describing function.

The development of an effective pursuit display which does not culminate in an effective precognitive display can be expected to be found in situations where the input function has internal coherence,

[4] See Elkind (**10**) and McRuer and Krendel (**3** or **4**) for details on the curve-fitting procedure which determined Y_p.

[5] Restrictions of the active human transfer elements to Y_{pi} and Y_{pc} is not a crucial step for Elkind's explanation, but is merely a simplification. The results can also be explained in a similar fashion with Y_{pi} and Y_{pr} both active.

but is not perfectly predictable. An example of such an input would be a carrier signal with a superimposed modulation. The ease with which a human operator can detect the carrier frequency depends on the relative amplitudes and frequencies of the modulating wave

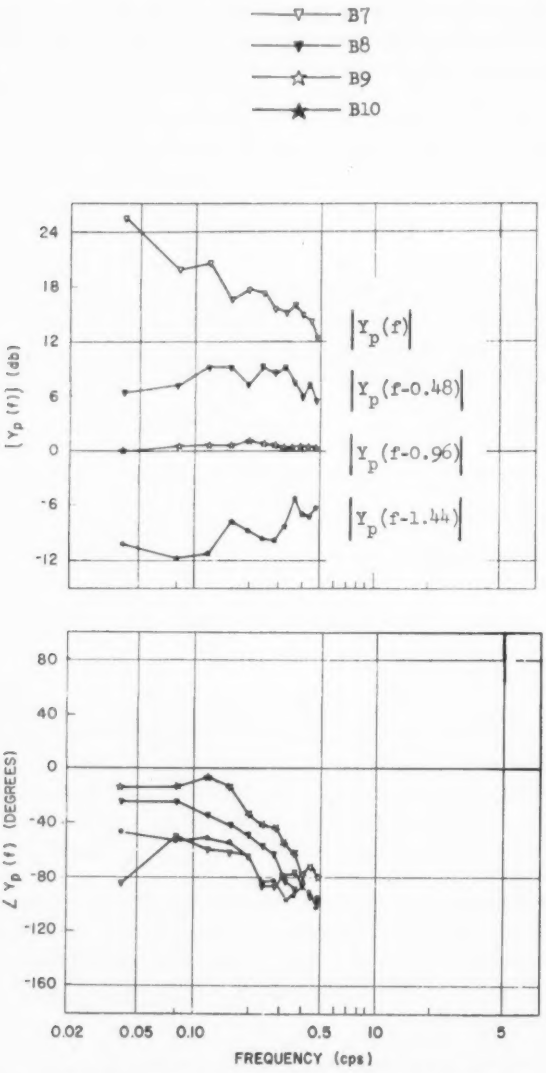


FIG. 8. Elkind's bandpass forcing function data.

forms. In a tracking task effective pursuit-type behavior would be evidenced if the operator's response revealed that he had, indeed, recognized the carrier frequency.

Elkind's data generated with rectangular bandpass spectra forcing functions and a compensatory display provide just such evidence. The form of the forcing functions used is shown in Fig. 7. Phenomenally, these forcing functions resemble a varying carrier frequency on which noise has been superimposed. The open loop describing functions for these forcing functions are presented in Fig. 8. The data for bandpass spectra B8, B9, and B10 were translated down the frequency scale so that the lowest frequency component of the forcing function corresponded to the origin on the new frequency scale. To some extent, these translated transfer functions are similar to those associated with the normal low-pass inputs. In other words, the human operator appears to be able to shift his low-pass characteristics up the frequency scale, acting like a carrier servo system, when he is confronted with a bandpass input. When he makes these transitions, his gain is reduced and phase lag increased. Clearly, however, the human is able to extract and operate upon the fact that the stimuli contain frequencies starting at some value other than zero by effectively responding to these starting frequencies.

Using the type of explanation illustrated in Fig. 4 for this instance, we would say that the physical compensatory display is replaced by an "effective" pursuit situation where the extra "blocks" in the block diagram are generated internally as a result of the forcing function's coherence.

B. Precognitive Organization

In the preceding section we have shown how a pursuit organization may be imposed by the operator for a partially coherent input signal. Clearly the ideal laboratory demonstration of precognitive functioning would require a perfectly predictable input function, such as a simple sinusoid. We would expect the tracking performance to proceed as indicated by Figs. 3, 4 and 5. Just such an example has been provided by Mayne (11).

The experimental situation was one in which subjects were to follow a line on moving recording paper which could be viewed through a narrow, transverse slot. The original line drawn on the paper served as the forcing function. A line drawn by the subject was the output.

The results are typical of those reported by Mayne (11) and Ellson and Gray (12) among others. The most interesting results are illustrated in Fig. 9.

1. The initial response under a *compensatory* organization (not shown in Fig. 9) contains a reaction time delay.

2. The second phase (Fig. 9*a*) reveals the presence of a smaller phase lag than before. In this pursuit organization the phase lag is too small to be due to a reaction-time delay. (A factor of $e^{-\tau s}$ in the transfer function would give a phase lag of 90° at a frequency of 1 cps if τ is taken to be 0.25 sec.)

3. Finally, when the completely predictable sinusoid has been "recognized" the operator manifests a *precognitive* organization and the response changes to a "synchronous"[6] following of the sine wave with no phase lag and only slight attenuation. This mode can be maintained fairly easily up to frequencies of 2.5 to 3 cps. (It should be noted that similar synchronous modes also exist for other forcing functions, such as square waves.)

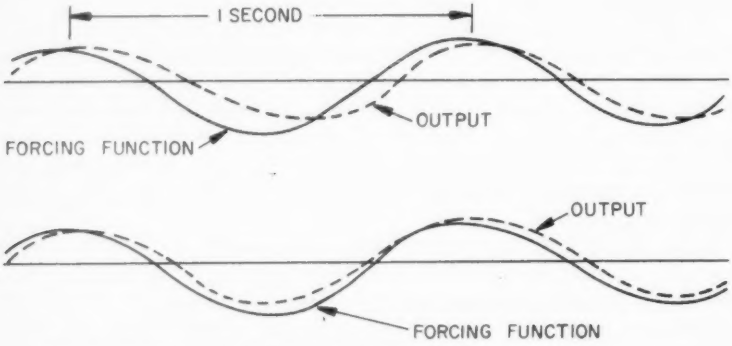


FIG. 9. Typical parts of a closed loop response (at top) and a synchronous response to a simple sinusoid.

After a period in the synchronous mode, the operator tends to drift out of synchronism. When this occurs, he gets back into the synchronous mode by going through the first two response phases noted above, though usually more rapidly than the first time.

It is of interest to note that the pursuit organization interpretation of Fig. 9*a*—that is, the operator's compensation for the phase lag due to $e^{-\tau s}$—is supported by other related research by Mayne. Using the same apparatus he examined responses to step inputs. He noted two phases to the subject's response. The first phase consists of a dead portion (reaction-time delay) during which the subject makes no movement at all. After this initial period there is the *dynamic portion* of the response, during which the subject moves relatively quickly, ending with a small error (the primary movement of this positioning response), and then sometimes more slowly toward the new position of the line, eliminating the remaining error (the secondary movement).

[6] Mayne (11) introduced the term "synchronous" to describe what we have chosen to call precognitive behavior. The term "precognitive" was introduced by North (13).

Mayne suggests that the dynamic portion of the response to a step function can be correlated with the closed loop response to a sine wave. In this connection, it is worth noting that the 40° phase lag observed in Fig. 9*a* is approximately equal to the phase lag at 1 cps which would be caused by a transfer function deduced from the "dynamic" portion of the response to a step input. Thus the e^{-rs} lag is eliminated, just as it was in the Elkind pursuit data presented in Fig. 6*b*.

VII. IMPLICATIONS OF THE SUCCESSIVE ORGANIZATIONS OF PERCEPTION MODEL

A. Future Experiments

In the foregoing sections we have demonstrated the compatibility of the constructs of the SOP model with such data as were available in the literature. Since the data which we have gleaned were not generated with the testing of the SOP model in mind, there are areas of interest for which we lack empirical data. A series of directly pertinent experiments is desirable to explore these areas. These experiments should serve to answer the following general questions:

1. What measures can be imposed on input signal coherence?
2. By what other means may human progress from compensatory to pursuit to precognitive organization be demonstrated and functionally related to a measure of input signal coherence?
3. How long does it take to establish the developmental stages in the SOP model for inputs of given coherence, and how stable are these developmental stages?
4. How do controlled element dynamics of other than unity interract with the SOP model?
5. What are the implications of the SOP model for training procedures?

Considering the first question, an experimental measure of signal coherence is implied by the comparisons between the Elkind and Mayne work illustrated in Figs. 8 and 9. The pure sinusoid of the Mayne example can be thought of as the output for a white noise input to an idealized bandpass filter which passes only one narrow frequency band and has infinitely steep skirts. By broadening the pass band of this filter, or decreasing the slope of the skirts, we can generate an input signal which is decreasingly coherent in a manner defined by the filter shape.

Another means for approaching this problem is to start with a sine wave and to add successively more and more sinusoids of given frequencies, amplitudes and phases. In this case a random signal is approached as we achieve a summation of sine waves of random phases.

Regarding the second question, in this paper a major indication of developmental status in the SOP model is the phase lag. Clearly there

are a great many detailed experiments which can be performed relating a ranking of inputs as implied in the foregoing paragraph with a measured phase lag. Phase-lag measurements, however, are difficult to achieve in general, and alternative measurement techniques should be considered. Since the basic human behavior which we are examining is the ability to act on signal coherence and to predict, one could use a visual signal blanking technique at various scheduled blanking rates and a simple performance criterion, such as rms error, to determine how well the subject had succeeded in predicting signals of different degrees of coherences.

The third and fourth questions are essentially detailed applications of techniques such as the foregoing in a broad series of experiments.

The fifth question also implies a broad series of experiments related to such areas as transfer of training and knowledge of results. From the foregoing development we would postulate, for example, devices which serve to make the subject more easily aware of the predictable nature of his display will serve to speed up his progress through the stages of the SOP model. Annett and Kay (2) have made a similar prediction and noted that the information value of signals changes during the course of skill development, making additional signals or "augmented feedback" useful during initial training.

Although very little has been stated explicitly about the extension of the SOP model to general motor learning, it is expected that such an extension can be accomplished most readily within the context of training studies.

B. Systems Design and Engineering Psychology

In the foregoing sections we have illustrated forms of external organization imposed on the stimuli by the experimenter. We have then presented evidence that a similar organization of the stimuli was performed by the subject. Such external organizations of the stimuli can be described in terms of servomechanism theory so that experimentally demonstrable characteristics of training such as decreased response latencies and increased response precision are derivable from these descriptions and are consistent with observed facts. That linear human describing functions are consistent with our hypothesized SOP model for motor skills does not mean that the hypothesized organization of the stimuli is dependent on a linear human operator. The linear human operator is a convenience, not a necessity, in our hypothesis for skill development.

The SOP model, properly elaborated, provides a basis for specifying both the process by which a human operator develops a motor skill as well as the *ultimate* level of proficiency which can be attained. The

question of the relative advantages of manual control with "conventional" displays, manual control with "unconventional" displays, and automatic control has become one of the central problems of engineering psychology today. The SOP model provides a useful position from which to attack this problem. Ultimately we hope to be able to specify trade-off functions of trained men *versus* complicated machines in terms of system criteria. To do this we must have definitive data on trainees available, training time, proficiencies achieved, and equipment reliability. When this goal has been accomplished, system designers will be able to use man to the full extent of his unique adaptive and predictive capabilities.

LIST OF SYMBOLS

$c(t)$ operator output
$C(s)$ transform of operator output
$E(s)$ transform of error
f_{co} cutoff frequency
T_1, T_2 operator's lag-time constant
$H(s)$ closed loop transfer function
$i(t)$ forcing function
$I(s)$ transform of system forcing function
K human operator transfer function gains
$n_c(t)$ operator remnant, open loop
$N_c(s)$ transform of open loop remnant
$r(t)$ system output
s Laplace operator
t time
y_p operator weighting function (compensatory)
y_{pi}, y_{pe}, y_{pr} operator weighting function (pursuit)
$Y_p(s)$ operator describing function (compensatory)
Y_{pi}, Y_{pe}, Y_{pr} operator describing function (pursuit)
$Y_c(s)$ controlled element transfer function
$\epsilon(t)$ system error
τ reaction-time delay

REFERENCES

(1) J. ANNETT AND H. KAY, "Skilled Performance," *Occupational Psychology*, Vol. 30, pp. 112–117 (1956).

(2) J. ANNETT AND H. KAY, "Knowledge of Results and Skilled Performance," *Occupational Psychology*, Vol. 31, pp. 70–79 (1957).

(3) D. T. McRUER AND E. S. KRENDEL, "Dynamic Response of Human Operators," Wright Air Development Center, WADC TR-56-524, October, 1957.

(4) D. T. McRUER AND E. S. KRENDEL, "The Human as a Component in a Servo System," JOUR. FRANKLIN INST., Vol. 267, pp. 381, 511 (1959).

(5) E. C. POULTON, "Two Pointer and One Pointer Displays in Tracking," APU, Psychological Laboratory, Cambridge, England, APU 142/50, 1950.

(6) J. W. SENDERS AND M. CRUZEN, "Tracking Performance on Combined Compensatory and Pursuit Tasks," Wright Air Development Center, WADC TR-52-39, 1952.

(7) R. CHERNIKOFF, H. P. BIRMINGHAM AND F. V. TAYLOR, "A Comparison of Pursuit and Compensatory Tracking under Conditions of Aiding and No Aiding," *J. Exptl. Psych.*, Vol. 49, pp. 55–59 (1955).

(8) R. CHERNIKOFF, H. P. BIRMINGHAM AND F. V. TAYLOR, "A Comparison of Pursuit and Compensatory Tracking in a Simulated Aircraft Control Loop," *J. Appl. Psychol.*, Vol. 40, pp. 47–52 (1956).

(9) R. CHERNIKOFF AND F. V. TAYLOR, "Effects of Course Frequency and Aided Time Constant on Pursuit and Compensatory Tracking," *J. Exptl. Psych.*, Vol. 53, pp. 285–292 (1957).

(10) JEROME I. ELKIND, "Characteristics of Simple Manual Control Systems," Technical Report No. 111, MIT Lincoln Laboratory, April 6, 1956.

(11) ROBERT MAYNE, "Some Engineering Aspects of the Mechanism of Body Control," *Elec. Eng.*, Vol. 70, pp. 207–212 (1951).

(12) D. G. ELLSON AND F. GRAY, "Frequency Responses of Human Operators Following a Sine Wave Input," Memo Report MCREXD-694-2N, USAF AMC, 1948.

(13) J. D. NORTH, "The Human Transfer Function in Servo Systems," in "Automatic and Manual Control," edited by A. Tustin, New York, Academic Press, 1952.

HYDRAULIC ANALOGUE FOR ONE DIMENSIONAL UNSTEADY GAS DYNAMICS

BY

W. H. T. LOH[1]

SUMMARY

For two dimensional steady flow, the equations of continuity, momentum, and energy have been found by many authors to be identical for an irrotational isentropic flow of a perfect gas with specific heat ratio of 2.0 and an incompressible frictionless flow of water in an open horizontal channel of a rectangular cross-section. The present paper presents the results found by the writer at MIT in 1944, for one dimensional unsteady flow. This work shows that the equations of continuity, momentum, energy, waves, and wave propagation are also identical for an isentropic perfect gas flow having any specific heat ratio and an incompressible frictionless water flow in an open horizontal channel of a cross-section described by the equation $z = Cy^n$.

INTRODUCTION

Analogy comes from similarities of basic flow equations. For two dimensional steady flow, the equations of continuity, momentum, and energy were found by many authors (**1, 2, 3, 4**)[2] to be identical in mathematical forms for a flow of an irrotational isentropic perfect gas of specific heat ratio of 2.0 and an incompressible frictionless flow of water in an open horizontal channel of a rectangular cross-section; they are:[3]

	Gas Flow	Water Flow
Continuity:	$u'\frac{\partial \rho'}{\partial x'} + \rho'\frac{\partial u'}{\partial x'} + v'\frac{\partial \rho'}{\partial y'} + \rho'\frac{\partial v'}{\partial y'} = 0$	$u'\frac{\partial h'}{\partial x'} + h'\frac{\partial u'}{\partial x'} + v'\frac{\partial h'}{\partial y'} + h'\frac{\partial v'}{\partial y'} = 0$
Momentum:	$u'\frac{\partial u'}{\partial x'} + v'\frac{\partial v'}{\partial x'} = -\left(\frac{1}{K-1}\right)\frac{\partial p^{(K-1)/K}}{\partial x'}$	$u'\frac{\partial u'}{\partial x'} + v'\frac{\partial v'}{\partial x'} = -\frac{\partial h'}{\partial x'}$
	$u'\frac{\partial u'}{\partial y'} + v'\frac{\partial v'}{\partial y'} = -\left(\frac{1}{K-1}\right)\frac{\partial p^{(K-1)/K}}{\partial y'}$	$u'\frac{\partial u'}{\partial y'} + v'\frac{\partial v'}{\partial y'} = -\frac{\partial h'}{\partial y'}$
Energy:	$\frac{u^2}{u^2_{\text{Max}}} = 1 - \left(\frac{T}{T_0}\right)$	$\frac{u^2}{u^2_{\text{Max}}} = 1 - \left(\frac{h}{h_0}\right)$

Comparison of the corresponding equations shows the following analogous terms:

---

[1] Staff Engineer, Chance Vought Aircraft, Inc., Dallas, Texas.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] A list of symbols used appears at the end of the paper.

GAS FLOW		WATER FLOW	
Density ratio	ρ'	= Water-depth ratio h'	
Pressure ratio	$p'^{(K-1)/K}$	= Water-depth ratio h'	
Temperature ratio	T'	= Water-depth ratio h'	
Velocity of sound	$a(=\sqrt{KRT})$	= Wave velocity	$C(=\sqrt{gh})$
Mach number	u'	= Froude number	u'

NOTE: For purposes of analogy, in the momentum equations a necessary condition is that $\frac{1}{K-1} = 1$.

The restriction that $\frac{1}{K-1} = 1$ necessarily limits comparison of the flow of water to the flow of a fictitious gas having[4] a specific heat ratio of 2.0. Although no such gas is found in nature, the hydraulic analogy has proved very useful for studying two dimensional subsonic and supersonic flows in a qualitative manner.

One dimensional unsteady flow equations were examined early in 1944 by the writer (5), under the guidance of E. S. Taylor at MIT. It was found then that they are identical in mathematical form for flow of an irrotational isentropic perfect gas (with a specific heat ratio of any value) and for flow of an incompressible frictionless water in an open horizontal channel with a cross-section described by the equation $z = Cy^n$. At a later date, similar results were also obtained independently at New York University by Probstein and Hudson (6). Shapiro (7, 8) of MIT essentially described the writer's results. Following the writer's early approach to one dimensional unsteady flow, Szebehely and Whicker (9) of Virginia Polytechnic Institute generalized the hydraulic analogue to apply to the external steady flow case. For internal unsteady flow, Schweitzer (10, 11) of Pennsylvania State University applied the writer's findings to a research study in exhaust dynamics. Although the work has been widely applied by several authors, the complete analysis has not been published, except for some simple results without derivations (12). It is the purpose of this paper to present the analysis for those who are particularly interested in the one dimensional unsteady flow hydraulic analogue.

BASIC ASSUMPTIONS

1. The fluid is frictionless so that conversion of mechanical energy into heat is excluded both in gas and in water.

2. The flow is one dimensional and is in a duct (for gas) or channel (for water) of uniform cross-section. This implies that the traverse components of fluid velocity are negligible compared with the axial component.

3. The viscosity and the surface tension of the water are neglected. In the analysis of waves, they are considered to be long waves so that the

[4] Similar results hold for two dimensional unsteady flow.

change of elevation of water is considered to be small in comparison with h_0, the water depth at equilibrium position.

4. The vertical acceleration of the water is negligible compared with the acceleration of gravity. Under this assumption the static pressure at a point in the field of flow is assumed to depend linearly on the vertical distance under the free surface at that position. In other words, $p = \rho g(h - y)$. It is further assumed that the velocity is uniform and constant over any cross-section perpendicular to the flow direction. Justification for this assumption for the case of one dimensional unsteady flow is given in Appendix A.

5. In one dimensional unsteady flow of gas dynamics, all parameters (pressure, temperature, velocity) are assumed to be uniform and constant across any section perpendicular to the direction of flow.

ANALYSIS

The well-known equations of continuity, momentum, and energy, in non-dimensional form, for one dimensional unsteady gas dynamics are:

$$u' \frac{\partial \rho'}{\partial x'} + \rho' \frac{\partial u'}{\partial x'} + \left(\frac{l_a}{a_0 t_a}\right) \frac{\partial \rho'}{\partial t'} = 0$$

$$\frac{\partial u'}{\partial t'} + \left(\frac{a_0 t_a}{l_a}\right) u' \frac{\partial u'}{\partial x'} = - \left(\frac{a_0 t_a}{l_a}\right) \left(\frac{1}{K-1}\right) \frac{\partial p'^{(K-1)/K}}{\partial x'}$$

$$\frac{\partial T'}{\partial t'} + \left(\frac{a_0 t_a}{l_a}\right) u' \frac{\partial T'}{\partial x'} + (K-1) p' \frac{\partial \left(\frac{1}{\rho'}\right)}{\partial t'} + (K-1) \left(\frac{a_0 t_a}{l_a}\right) p' u' \frac{\partial \left(\frac{1}{\rho'}\right)}{\partial x'} = 0$$

Since analogy comes from the similarities of basic flow equations, the analysis here is limited to:

1. the derivation of equations of continuity, momentum, and energy for the corresponding water flow, and

2. the comparison of their similarities with corresponding gas dynamics equations.

DERIVATIONS

The derivations given below are for an incompressible frictionless water flow in an open horizontal channel with a constant cross-section described by the equation $z = Cy^n$.

1. Continuity

According to the continuity equation, the net mass rate of flow crossing the control surfaces must be equal to the net rate of change of

mass inside the control volume. At a given instant, the mass rate of flow into the left boundary is ρAu and the mass within the control volume is $\rho A dx$. Hence one may write:

$$-\frac{\partial(\rho Au)}{\partial x}dx = \frac{\partial}{\partial t}(\rho A dx).$$

Expanding this and noting that ρ is constant for water flow, one obtains:

$$u\frac{\partial A}{\partial x} + A\frac{\partial u}{\partial x} + \frac{\partial A}{\partial t} = 0.$$

Putting into non-dimensional form and noting that

$$A = \int_0^h Cy^n dy = C\left(\frac{h^{n+1}}{n+1}\right),$$

one obtains:

$$u'\frac{\partial h'^{n+1}}{\partial x'} + h'^{n+1}\frac{\partial u'}{\partial x'} + \left(\frac{l_w}{\sqrt{\frac{gh_0}{n+1}}\,t_w}\right)\frac{\partial h'^{n+1}}{\partial t'} = 0.$$

2. Momentum

Consider the small fluid volume between sections $(a-a)$ and $(b-b)$ which are dx apart, as one which is generated by moving the plane section of area A_x perpendicular to the x-axis from x to $x+dx$. Newton's law, as it applies to this fluid volume, says that the net force on it is equal to mass times acceleration: The net force is the difference between pressure forces on section $(a-a)$ and section $(b-b)$, and is equal to:

$$F_{a-a} - F_{b-b} = \int_0^h \rho g(h-y)z\,dy - \int_0^{h+(\partial h/\partial x)dx} \rho g(h-y)z\,dy$$

$$= -\frac{\rho g}{(n+1)(n+2)}\left[h^{n+2} - \left(h + \frac{\partial h}{\partial x}dx\right)^{n+2}\right].$$

The mass between sections $(a-a)$ and $(b-b)$ may easily be obtained after assuming the surface change between the sections to be linear and therefore equal to:

$$\rho\left[h^{n+2} - \left(h + \frac{\partial h}{\partial x}dx\right)^{n+2}\right]\frac{1}{\dfrac{\partial h}{\partial x}(n+1)(n+2)}.$$

The acceleration in terms of partial derivatives is:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}.$$

By Newton's law as stated above, one obtains, after simplification in non-dimensional form:

$$\frac{\partial u'}{\partial t'} + \left(\frac{\sqrt{\frac{gh_0}{n+1}}\,t_w}{l_w}\right)u'\frac{\partial u'}{\partial x'} = -\left(\frac{\sqrt{\frac{gh_0}{n+1}}\,t_w}{l_w}\right)(n+1)\frac{\partial h'}{\partial x'}.$$

3. Energy

The law of conservation of energy requires that the difference in the rate of supply energy entering a control volume and the rate at which energy leaves it must be equal to the net rate of increase of energy in it. Using the same control volume as specified in the derivation of the momentum equation, one obtains immediately:

Rate of energy entering at $(a - a)$

$$= \int \tfrac{1}{2}u^2\rho u dA + \int (gy)\rho u dA + \int p u dA$$

Rate of energy leaving at $(b - b)$

$$= \left[\int \tfrac{1}{2}u^2\rho u dA + \int (gy)\rho u dA + \int p u dA\right] + \frac{\partial}{\partial x}\left[\int \tfrac{1}{2}u^2\rho u dA + \int (gy)\rho u dA + \int p u dA\right]dx$$

Rate of energy increase in control volume between sections $(a - a)$ and $(b - b)$

$$= \frac{\partial}{\partial t}\left[\int \tfrac{1}{2}u^2\rho dA dx + \int (gy)\rho dA dx\right]$$

Therefore the energy equation, according to the law of conservation of energy as stated above, is:

$$-\frac{\partial}{\partial x}\left[\int \tfrac{1}{2}u^2\rho u dA + \int (gy)\rho u dA + \int p u dA\right]dx = \frac{\partial}{\partial t}\left[\int \tfrac{1}{2}u^2\rho dA + \int (gy)\rho dA\right]dx$$

Expanding it and noting that $dA = Cy^n dy$, $p = \rho g(h - y)$, one obtains after simplification in non-dimensional form:

$$\frac{\partial h'}{\partial t'} + \left(\frac{\sqrt{\frac{gh_0}{n+1}}\, t_w}{l_w}\right) u' \frac{\partial h'}{\partial x'} + \left(\frac{h'^{n+2}}{n+1}\right) \frac{\partial \left(\frac{1}{h'^{n+1}}\right)}{\partial t'} + \left(\frac{h'^{n+2}}{n+1}\right)\left(\frac{\sqrt{\frac{gh_0}{n+1}}\, t_w}{l_w}\right) u' \frac{\partial \left(\frac{1}{h'^{n+1}}\right)}{\partial x'} = 0.$$

COMPARISONS

Summarizing all the equations, one has:

GAS FLOW

Continuity:

$$u' \frac{\partial \rho'}{\partial x'} + \rho' \frac{\partial u'}{\partial x'} + \left(\frac{l_a}{a_0 t_a}\right) \frac{\partial \rho'}{\partial t'} = 0$$

Momentum:

$$\frac{\partial u'}{\partial t'} + \left(\frac{a_0 t_a}{l_a}\right) u' \frac{\partial u'}{\partial x'} = -\left(\frac{a_0 t_a}{l_a}\right)\left(\frac{1}{K-1}\right) \frac{\partial p'^{(K-1)/K}}{\partial x'}$$

Energy:

$$\frac{\partial T'}{\partial t'} + \left(\frac{a_0 t_a}{l_a}\right) u' \frac{\partial T'}{\partial x'} + (K-1) p' \frac{\partial \left(\frac{1}{\rho'}\right)}{\partial t'} + (K-1)\left(\frac{a_0 t_a}{l_a}\right) p' u' \frac{\partial \left(\frac{1}{\rho'}\right)}{\partial x'} = 0$$

WATER FLOW

Continuity:

$$u' \frac{\partial h'^{n+1}}{\partial x'} + h'^{n+1} \frac{\partial u'}{\partial x'} + \left(\frac{l_w}{\sqrt{\frac{gh_0}{n+1}}\, t_w}\right) \frac{\partial h'^{n+1}}{\partial t'} = 0$$

Momentum:

$$\frac{\partial u'}{\partial t'} + \left(\frac{\sqrt{\frac{gh_0}{n+1}}\, t_w}{l_w}\right) u' \frac{\partial u'}{\partial x'} = -\left(\frac{\sqrt{\frac{gh_0}{n+1}}\, t_w}{l_w}\right)(n+1) \frac{\partial h'}{\partial x'}$$

Energy:

$$\frac{\partial h'}{\partial t'} + \left(\frac{\sqrt{\frac{gh_0}{n+1}}\, t_w}{l_w}\right) u' \frac{\partial h'}{\partial x'} + \left(\frac{1}{n+1}\right) h'^{n+2} \frac{\partial \left(\frac{1}{h'^{n+1}}\right)}{\partial t'} + \left(\frac{1}{n+1}\right)\left(\frac{\sqrt{\frac{gh_0}{n+1}}\, t_w}{l_w}\right) h'^{n+2} u' \frac{\partial \left(\frac{1}{h'^{n+1}}\right)}{\partial x'} = 0$$

A comparison of these basic equations shows that they are identical in mathematical form; therefore an analogy exists in one dimensional unsteady flow between an isentropic perfect gas flow having any value of specific heat ratio and an incompressible frictionless water flow in

an open horizontal channel with a cross-section described by the equation $z = Cy^n$. The analogous quantities are as follows:

local speed of sound $a = \sqrt{KRT}$	$\sim$ local speed of wave propagation $C = \sqrt{\frac{gh}{n+1}}$
local Mach number $M = \frac{u}{a}$	= local Mach number $M = \frac{u}{C}$
local density ratio ρ'	= local water-depth ratio h'^{n+1}
local pressure ratio p'	= local water-depth ratio h'^{n+2}
local temperature ratio T'	= local water-depth ratio h'
flow similarity number $\left(\frac{a_0 l_a}{l_a}\right)$	= flow similarity number $\left(\frac{\sqrt{\frac{gh_0}{n+1}}\, t_w}{l_w}\right)$
function of gas specific heat ratio $\left(\frac{1}{K-1}\right)$	= function of channel cross-section exponent $(n+1)$

The last condition must be fulfilled in order for the water channel to represent a gas flow of desired specific heat ratio K. For ordinary gas dynamics problems, the following channel cross-sections are useful:

1. Parabolic	$n = 1.5$	$K = 1.4$
2. Triangular	$n = 1.0$	$K = 1.5$
3. Rectangular	$n = 0$	$K = 2.0$

HYDRAULIC ANALOGY FOR LONGITUDINAL PLANE WAVES

To clarify the hydraulic analogy to a greater degree for the case of one dimensional unsteady flow in ducts and in channels, it is best to check further the mathematical similarities of fundamental wave equations. Only the simple equations of wave formation produced by simple causes are considered here; otherwise, the equations would be too complicated and difficult to handle. However, it is believed that the analogy of plane waves, which exists in these simple equations, will probably also exist in the complicated wave mechanisms. Based on this point of view, the following analysis was made. The gas wave is considered to be set up in a duct. The water wave is considered to be set up in an open horizontal channel with a cross-section described by the equation $z = Cy^n$. In both cases the duct and the channel have a constant cross-section. The waves considered are plane waves and are set up in the fluid medium, while the fluid is considered to be motionless except for small oscillations about the equilibrium positions. In both cases, the restoring force responsible for keeping the wave going is simply the opposition that the medium exhibits against being compressed.

WAVES IN GAS

The basic equations of longitudinal plane waves traveling in a constant area duct are well known (13); they are:

$$\rho'\left(1+\frac{\partial\xi}{\partial x}\right)=1$$

$$\frac{\partial^2\xi}{\partial t^2}=-\frac{1}{K}a_0^2\frac{\partial}{\partial x}[p'-1]$$

$$\frac{\partial^2\xi}{\partial t^2}=a_0^2\frac{\partial^2\xi}{\partial x^2}$$

$$\frac{\partial^2}{\partial t^2}[p'-1]=a_0^2\frac{\partial^2}{\partial x^2}[p'-1]$$

$$\frac{\partial^2}{\partial t^2}[\rho'-1]=a_0^2\frac{\partial^2}{\partial x^2}[\rho'-1].$$

WAVES IN WATER

The corresponding equations of longitudinal plane waves travelling in an open channel with a constant cross-section are derived as follows: Consider the frictionless water to be in an open channel with uniform cross-section A. When an infinitesimal plane disturbance wave passes through the channel, the planes at different positions along the channel will be displaced from their equilibrium positions back and forth along the channel, with the displacement depending on both t and x. The water ahead of one plane will always be ahead of that plane, and the water between two planes will always be between those two planes (Appendix A). Thus, water particles on one plane will remain on that plane when the disturbance wave penetrates. Now examine two planes which at equilibrium are at the distances x and $x+dx$. The water between them has a depth h_0, so the volume between them is $Adx = C\frac{{h_0}^{n+1}}{n+1}dx$. When the planes are slightly displaced to a new position at time t, the volume between the planes will not be changed; but the distance between the planes is changed due to the fact that the displacement of one plane is $\xi(x)$ and that of the other is $\xi(x+dx) = \xi(x)+\frac{\partial\xi}{\partial x}dx$. The distance between the planes at time t is

$$dx+\xi(x+dx)-\xi(x)=dx+\frac{\partial\xi}{\partial x}dx.$$

The water depth between the planes must be changed to h so that the total volume between them can remain unchanged.

$$\left(\frac{{h_0}^{n+1}}{n+1}\right)dx=\left(\frac{h^{n+1}}{n+1}\right)\left(dx+\frac{\partial\xi}{\partial x}dx\right).$$

After simplification, the equation reduces to:

$$h'^{n+1}\left(1+\frac{\partial \xi}{\partial x}\right)=1. \tag{1}$$

The equation of motion gives:

$$\rho C\left(\frac{h_0{}^{n+1}}{n+1}\right)dx\,\frac{\partial^2\xi}{\partial t^2}=-\frac{\partial F}{\partial x}dx$$

where ξ is the displacement and $\frac{\partial^2\xi}{\partial t^2}$ is the acceleration. $\frac{\partial F}{\partial x}dx$ is the net force, and is equal to

$$\frac{\partial}{\partial x}\int_0^h \rho g(h-y)z\,dy\,dx=\frac{\rho g C}{n+1}h^{n+1}\frac{\partial h}{\partial x}dx.$$

After simplification, the equation of motion reduces to:

$$\frac{\partial^2\xi}{\partial t^2}=-\left(\frac{n+1}{n+2}\right)\left(\frac{gh_0}{n+1}\right)\frac{\partial}{\partial x}[h'^{n+2}-1]. \tag{2}$$

Eliminating h' in Eqs. 1 and 2, one obtains:

$$\frac{\partial^2\xi}{\partial t^2}=\left(\frac{gh_0}{n+1}\right)\frac{\partial^2\xi}{\partial x^2}\left[\frac{1}{\left(1+\frac{\partial\xi}{\partial x}\right)^{(2n+3)/(n+1)}}\right].$$

Noting that $\frac{\partial\xi}{\partial x}$ is small in comparison to 1, one obtains:

$$\frac{\partial^2\xi}{\partial t^2}=\left(\frac{gh_0}{n+1}\right)\frac{\partial^2\xi}{\partial x^2}. \tag{3}$$

Combining Eqs. 1 and 3, one obtains:

$$\frac{\partial^2}{\partial t^2}[h'^{n+2}-1]=\left(\frac{gh_0}{n+1}\right)\frac{\partial^2}{\partial x^2}[h'^{n+2}-1]$$

$$\frac{\partial^2}{\partial t^2}[h'^{n+1}-1]=\left(\frac{gh_0}{n+1}\right)\frac{\partial^2}{\partial x^2}[h'^{n+1}-1].$$

Summarizing all the wave equations, one has:

LONGITUDINAL PLANE WAVES IN GAS	LONGITUDINAL PLANE WAVES IN WATER
$\rho'\left(1+\frac{\partial\xi}{\partial x}\right)=1$	$h'^{n+1}\left(1+\frac{\partial\xi}{\partial x}\right)=1$
$\frac{\partial^2\xi}{\partial t^2}=-\frac{1}{K}a_0^2\frac{\partial}{\partial x}[p'-1]$	$\frac{\partial^2\xi}{\partial t^2}=-\left(\frac{n+1}{n+2}\right)\left(\frac{gh_0}{n+1}\right)\frac{\partial}{\partial x}[h'^{n+2}-1]$
$\frac{\partial^2\xi}{\partial t^2}=a_0^2\frac{\partial^2\xi}{\partial x^2}$	$\frac{\partial^2\xi}{\partial t^2}=\left(\frac{gh_0}{n+1}\right)\frac{\partial^2\xi}{\partial x^2}$
$\frac{\partial^2}{\partial t^2}[p'-1]=a_0^2\frac{\partial^2}{\partial x^2}[p'-1]$	$\frac{\partial^2}{\partial t^2}[h'^{n+2}-1]=\left(\frac{gh_0}{n+1}\right)\frac{\partial^2}{\partial x^2}[h'^{n+2}-1]$
$\frac{\partial^2}{\partial t^2}[\rho'-1]=a_0^2\frac{\partial^2}{\partial x^2}[\rho'-1]$	$\frac{\partial^2}{\partial t^2}[h'^{n+1}-1]=\left(\frac{gh_0}{n+1}\right)\frac{\partial^2}{\partial x^2}[h'^{n+1}-1]$

A comparison of these basic wave equations shows that they are identical in mathematical form; therefore an analogy exists between longitudinal plane waves in ducts (for gas) and longitudinal plane waves in channels (for water). The analogous quantities are consistent with those found previously.

HYDRAULIC ANALOGY FOR SPEED OF SOUND

In the previous derivations the correspondence of the velocity of sound, $a=\sqrt{KRT}$, in gas dynamics and the term, $C=\sqrt{\frac{gh}{n+1}}$, in hydraulic analogy was noted. The physical significance of the term $C=\sqrt{\frac{gh}{n+1}}$ is obtained by noting that the velocity of propagation of gravity waves of water is equal to $\sqrt{\frac{gA}{b}}$ (14). Here A is the cross-sectional area and b is the breadth of the water free surface. For the present case A is equal to $C\left(\frac{h^{n+1}}{n+1}\right)$ and b is equal to Ch^n. Therefore the velocity of propagation of long gravity waves of water in the present case is:

$$\sqrt{\frac{gA}{b}}=\sqrt{\frac{gh}{n+1}}=C.$$

Because the velocity in a water channel can be made as small as one thousandth or less of that occurring in a gas stream, according to the analogous relationship

$$\text{velocity of sound}=\sqrt{K\frac{p}{\rho}}\sim\text{wave propagation velocity}=\sqrt{\frac{gh}{n+1}},$$

the time scale can correspondingly be lengthened. This makes the hydraulic analogue method a relatively easy one for observation and

study of transient effects in gas dynamics. The slow motion can even be seen by human eyes. This feature is particularly useful for the study of unsteady gas dynamics in ducts.

SYMBOLS

a = local velocity of sound
C_P = specific heat at constant pressure
C_V = specific heat at constant volume
C = a constant or velocity of propagation of long gravity waves of water
g = gravitational constant
h = local water depth
K = ratio of specific heats = C_P/C_V
l = some characteristic length
n = exponent in relation $z = Cy^n$
p = local pressure
t = time or some characteristic time
T = local temperature
u = local velocity in x direction
x, y, z = rectangular coordinates, x in the flow direction, y in the vertical direction and z in the horizontal direction
ρ = local density
ξ = a function of x, which represents the displacement of the plane disturbed by the plane wave motion

Subscripts

a = air flow
w = water flow
$_0$ = undisturbed condition or the initial equilibrium condition
$'$ = prime is used to represent the non-dimensional quantity of the parameter; for example, $\rho' = \rho/\rho_0$, $p' = p/p_0$ etc. u', t', x', however, are defined as u/a_0 or u/C_0, t/t_a, or t/t_w, x/l_a or x/l_w, respectively.

REFERENCES

(1) E. PREISWERK, "Application of the Methods of Gas Dynamics to Water Flows With Free Surfaces," NACA TM-934 and 935.
(2) A. T. IPPAN AND R. T. KNAPP, "Study of High Velocity Flow in Curved Channels of Rectangular Cross-section," *Trans. Am. Geoph. Union*, 1936.
(3) D. RIABOUCHINSKY, "Sur l'analogie Hydraulique des Mouvements d'un fluide compressible," *Compt. Rend. de l'Acad. des Sciences*, Vol. 195, pp. 998–999 (1932).
(4) T. VON KARMAN, "Eine praktische Anwendung der Analogie Zwischen Uberschall-Stromung in Gasen und Uberkritesder Stromung in offenen Gerinnen," *Z.A.M.M.*, Vol. 18–1, pp. 49–56 (1938).
(5) W. H. T. LOH, "A Study of the Dynamics of the Induction and Exhaust Systems of a Four Stroke Engine by a Hydraulic Analogy," Sc.D. Thesis, MIT, 1946.

(6) R. F. PROBSTEIN AND G. E. HUDSON, "A Water Analogue of the Isentropic Flow of Compressible Gases which Have Arbitrary Ratios of Specific Heats," Project SQUID Report No. NYU-20-R, August 1948.
(7) A. H. SHAPIRO AND L. MALAVARD, "Analogue Methods in Physical Measurements in Gas Dynamics and Combustion," in "High Speed Aerodynamics and Jet Propulsion," Vol. IX, Princeton, Princeton University Press, 1954, pp. 309–321.
(8) A. H. SHAPIRO, "An Appraisal of the Hydraulic Analogue to Gas Dynamics," Massachusetts Institute of Technology Meteor Report 34, 1949.
(9) V. G. SZEBEHELY AND L. F. WHICKER, "Generalization and Improvement of the Hydraulic Analogy," *Proc. 1st. U.S. Nat'l Congress Applied Mech., 1951*, Ann Arbor, Edwards Brothers, Inc., 1952, pp. 893–901.
(10) P. H. SCHWEITZER, "Research in Exhaust Manifolds," *Trans. ASME*, Vol. 74, pp. 517–528 (1952).
(11) P. H. SCHWEITZER AND T. C. TSU, "Improving Engine Performance by Exhaust Pipe Tuning," ONR Report, 1951.
(12) W. H. T. LOH, "Hydraulic Analogy for Two Dimensional and One Dimensional Flows," *J. Aero-Space Sci.*, Vol. 26, pp. 389–390 (1959).
(13) P. M. MORSE, "Vibration and Sound," New York, McGraw-Hill Book Co., Inc., 1936.
(14) J. MCCOWAN, *Phil. Mag.*, Vol. 33, p. 258 (1892).

APPENDIX A

Discussion on the Assumption that the Velocity Is Constant over the Entire Cross-section

Consider the surface S, which encloses a volume V, fixed in space. The law of conservation of energy requires that the difference in the rate of supply of energy to V and the rate at which energy goes out through S must be equal to the net rate of increase of energy in V. Let U be the total energy per unit mass, then we have immediately:

$$-\int_S U\rho u_i n_i dS = \frac{\partial}{\partial t}\int_V U\rho dV.$$

By Green's theorem,

$$\int_S A n_i dS = \int_V \frac{\partial A}{\partial x_i} dV.$$

To transform the surface integral into the volume integral, then,

$$\int_v \left\{ -\frac{\partial}{\partial x_i}(U\rho u_i) - \frac{\partial}{\partial t}(U\rho) \right\} dV = 0.$$

Since V is arbitrarily chosen, the integrand itself must be zero. Therefore,

$$\frac{\partial}{\partial x_i}(U\rho u_i) + \frac{\partial}{\partial t}(U\rho) = 0.$$

Expanding and subtracting the continuity equation, we have

$$\frac{\partial}{\partial x_i}(U\rho u_i) + \frac{\partial}{\partial t}(U\rho) = \rho\left[\frac{\partial U}{\partial t} + u_i \frac{\partial U}{\partial x_i}\right] = 0.$$

In the present case of one dimensional unsteady incompressible flow, this becomes

$$\frac{\partial}{\partial t}\left[\frac{\rho}{2}u^2 + p + \rho g y\right] + u\frac{\partial}{\partial x}\left[\frac{\rho}{2}u^2 + p + \rho g y\right] = 0.$$

Now we assume that the vertical acceleration of the water is negligible compared with the acceleration of gravity, and that the pressure at a point in the field of flow depends on the vertical distance under the free surface at that point (assumption 4). In other words,

$$p = \rho g(h - y).$$

Substituting this into the previous equation, one obtains

$$\frac{\partial}{\partial t}\left[\rho\frac{u^2}{2}+\rho g(h-y)+\rho gy\right]+u\frac{\partial}{\partial x}\left[\rho\frac{u^2}{2}+\rho g(h-y)+\rho gy\right]=0$$

which is equal to:

$$\frac{\partial}{\partial t}\left(\frac{u^2}{2}+gh\right)+u\frac{\partial}{\partial x}\left(\frac{u^2}{2}+gh\right)=0.$$

The equation does not contain the coordinate y; therefore the velocity (the particle to be considered is, arbitrarily, y distance above the horizontal bottom) is constant over the entire depth of y. In other words, the velocity is constant over the entire cross-section and is a function of h only.

THE FRANKLIN INSTITUTE

Minutes of the Stated Meeting
December 16, 1959

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Mr. Wynn Laurence LePage, President, called the meeting to order with approximately 400 members and guests present.

The President stated that the minutes of the Stated Meeting of November 18, 1959 will be published in the next issue of the JOURNAL OF THE FRANKLIN INSTITUTE and will be presented at the next Stated Meeting of The Institute for approval.

The President announced that at the Annual Meeting of The Franklin Institute to be held on January 20, 1960 ten members are to be elected to the Board of Managers for a term of three years beginning in 1960, and that the Secretary had informed him that eleven nominees have been named and nominations were closed fifteen days before this Stated Meeting in accordance with the By-Laws. Mr. Richard T. Nalle, Chairman of the Nominating Committee, presented the following report:

"Under the provisions of Article VIII, Section 3 and Article IV, Section 5 of the By-Laws, the following eleven names have been properly placed in nomination to fill the ten vacancies on the Board of Managers which will occur at the Annual Meeting of The Institute on January 20, 1960:

"Mr. Brandon Barringer	Mr. Joseph Gray Jackson	Mr. S. Wyman Rolph
Mr. Edward G. Budd, Jr.	Mr. Ezra S. Krendel	Mr. John Russell, Jr.
Mr. J. Frank Cox	Mr. James McGowan, Jr.	Mr. W. Maxwell Scott, Jr.
	Mr. Richard T. Nalle	Dr. I. Melville Stein

"The Nominating Committee is required by the By-Laws to submit to the membership at this meeting its recommendations with respect to the candidates. Accordingly, the Nominating Committee recommends to the membership that the following ten gentlemen be elected to fill the vacancies which will exist. All the recommended nominees are currently members of the Board of Managers.

"Mr. Brandon Barringer	Mr. Joseph Gray Jackson	Mr. John Russell, Jr.
Mr. Edward G. Budd, Jr.	Mr. James McGowan, Jr.	Mr. W. Maxwell Scott, Jr.
Mr. J. Frank Cox	Mr. Richard T. Nalle	Dr. I. Melville Stein
	Mr. S. Wyman Rolph	

"The ballot will include all the nominees and the recommendations of this Committee will be indicated."

The President appointed the following Tellers of Election, who are neither officers nor nominees, in accordance with the By-Laws: Dr. Joseph S. Hepburn, Mr. John R. Kane and Mr. John H. Neher.

The President noted that in 1848, Elliott Cresson deeded to two trustees, a fund, from the income of which medals were to be made and presented by The Institute. The Deed of Trust further provided that successor trustees to the original two named in the Deed should be elected by the membership of The Institute. The present Trustees are Dr. Henry Butler Allen and Dr. John Frazer. However, Dr. Frazer has presented his resignation for reasons of health. On motion duly made and seconded, Dr. Frazer's resignation was accepted with heartfelt thanks for his long service as a Trustee of the Elliott Cresson Fund. The President presented the name of Rear Admiral Delmer S. Fahrney, U. S. Navy, Retired, the Secretary of the Committee on Science and the Arts, as Continuing Trustee of the Elliott Cresson Fund. On motion duly made and seconded, Rear Admiral Delmar S. Fahrney was elected.

The President noted that present with us tonight were our Silver Members and he asked that these twenty-five year members of The Institute stand. They were given warm applause.

The President announced that this Stated Meeting was also the occasion of the Charles S. Redding Lecture, established in 1955 in honor of Charles S. Redding, President of The Franklin Institute from 1942 to 1946 and Chairman of the Board of Leeds & Northrup Company at the time of his death, January 2, 1959. At this point he called upon Dr. Martin A. Pomerantz, Director of the Bartol Research Foundation of The Franklin Institute. Dr. Pomerantz presented Dr. W. F. G. Swann, Director Emeritus of the Bartol Research Foundation, with an appropriately engraved silver tray from the members of the staff of Bartol Research Foundation. Mr. LePage then presented to Dr. Swann a leather-bound folio of letters containing many congratulatory messages, all of which were unsolicited.

Mr. LePage then presented Dr. W. F. G. Swann, who for thirty-two years had been Director of the Bartol Research Foundation of The Franklin Institute, who gave an inspiring talk on "The Science of Yesterday, Today and Tomorrow."

At the conclusion of Dr. Swann's paper the audience gave thunderous applause and in adjourning the meeting the President expressed the warmth and gratitude of all present. The meeting adjourned at 9:50 P.M.

WILLIAM F. JACKSON, JR.
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, December 9, 1959.)

HALL OF THE COMMITTEE,
PHILADELPHIA, DECEMBER 9, 1959.

DR. WILLIAM G. SCHMIDT in the Chair.

The following report was presented for final action:

No. 3349: X-ray Image Amplifier.

This report recommended the award of an Edward Longstreth Medal to John W. Coltman, of Pittsburgh, Pennsylvania, "In consideration of his individual research contributions and of his guidance of the research program leading to the first working x-ray image-amplifier device."

D. S. FAHRNEY,
Secretary to Committee

MEMBERSHIP

The following elections were approved on December 4, 1959 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

Solomon Berg
Andrew H. Camerota
S. E. Charlton
W. B. Dean
D. F. Dickey
John C. Dorfman
F. J. Dunleavy
J. Maurice Dunne
Fred Kingsley Elder, Jr.
W. J. Fedechko
Arthur John Gehring, Jr.
George J. Hartman
William W. Jeanes
Louis Klein
Louis Lipschutz
John V. Martin
James J. McKee, Sr.
Matthew C. McKeown
Charles L. Metzger
Richard A. Mulford
Charles Robert Nelson
Josh T. Nessmith, Jr.
David B. Nibouar
Walter E. Nicholson, Jr.
James Y. Nishimura
C. V. O'Brien
Herbert L. Orlowitz
Percival S. Parkinson
Lemuel F. Parlette
Louise F. Peale
Richard Post
A. Hamilton Powell
John F. Preisendanz
Ralph H. Prichard
Joseph W. Reichenbach
Joseph John Reinhart
Henry E. Rhea
A. E. Rinckenberger
Stephen R. Rives
Earl W. Roberts, Jr.
James E. Robinson
R. F. Roehr
John S. Rogers
H. F. Schaefer
Robert J. Sigel
G. A. Smiley
A. F. Spangenberg
Bernard T. Svihel
Albert W. Volk
Wilson F. West
Albertus W. Whittaker
Edwin R. Wisner
Burton Woodside, Jr.

EDUCATOR

Joseph J. DeRenzis
Joanne Duncan
Curtis W. Gable
Robert Wayne Houston
M. J. Kotanchik
Joseph George Osterman
Harry M. Pfeffer

NON-RESIDENT

Robert David Doepken

NECROLOGY

Alfred J. Büchi, Sr. '53
Frank A. Craig '39
A. J. Creskoff '49
A. A. Furczyk '58
Mary K. Gibson '36
C. Hermann Krumbhaar, Jr. '36
Hiram S. Lukens '19
William P. Smith '58
Francis B. Vogdes '23

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 620 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays, and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

RECENT ADDITIONS

AERONAUTICS

Advances in Astronautical Sciences. Vol. 1–4. 1957–1959.
LEE, ASHER, ED. The Soviet Air and Rocket Forces. 1959.

ARCHITECTURE AND BUILDING

STERN, SYLVAN P. Elements of Reinforced Concrete. 1959.

ASTRONOMY

INTERNATIONAL ASTRONOMICAL UNION AND INTERNATIONAL SCIENTIFIC RADIO UNION. Paris Symposium on Radio Astronomy . . . held from 30 July to 6 August 1958 1959.
KOPAL, ZDENĚK. Close Binary Systems. 1959.

AUTOMATIC CONTROL

CALDWELL, WILLIAM I. ET AL. Frequency Response for Process Control. 1959.

CHEMISTRY AND CHEMICAL TECHNOLOGY

GOULD, DAVID F. Phenolic Resins. 1959.
KEULEMANS, ALOYSIUS IGNATIUS MARIE. Gas Chromatography. Ed. 2. 1959.
KLINE, GORDON MABEY, ED. Analytical Chemistry of Polymers. 1959.
LEVEY, MARTIN. Chemistry and Chemical Technology in Ancient Mesopotamia. 1959.
POPOVICH, MILOSH NICHOLAS AND HERING, CARL. Fuels and Lubricants. 1959.
SMOLIN, EDWIN M. AND RAPAPORT, LORENCE. s-Triazines and Derivatives. 1959.
Syntheses of Heterocyclic Compounds. Vol. 1. 1959.
Techniques of Polymer Characterization. 1959.
WHISTLER, ROY LISTER AND BEMILLER, JAMES N. Industrial Gums; Polysaccharides and their Derivatives. 1959.

DICTIONARIES

DEVRIES, LOUIS. German-English Science Dictionary; for Students in Chemistry, Physics, Biology, Agriculture and Related Sciences. Ed. 3. 1959.

ELECTRICITY AND ELECTRICAL ENGINEERING

CATTERMOLE, K. W. Transistor Circuits. 1959.
MOSES, MORRIS. Printed Circuits. 1959.

ENGINEERING

KINGERY, W. D. Property Measurements at High Temperatures; Factors Affecting and Methods of Measuring Material Properties at Temperatures above 1400° C (2550° F). 1959.
PRAGER, WILLIAM. An Introduction to Plasticity. 1959.

FOOD

MATZ, SAMUEL A., ED. The Chemistry and Technology of Cereals as Food and Feed. 1959.

GEOLOGY

GLAZOV, NIKOLAI VASILEVICH AND GLAZOV, ANATOLI NIKOLAEVICH. New Instruments and Methods of Engineering Geology. 1959.

HISTORY

DIDEROT, DENIS. A Diderot Pictorial Encyclopedia of Trades and Industry; Manufacturing and the Technical Arts in Plates Selected from "L'Encyclopédie ou Dictionnaire Raisonné des Sciences, des Arts et des Métiers." 1959.

MATHEMATICS

CHURCHMAN, CHARLES WEST AND RATOOSH, PHILBURN, ED. Measurement; Definitions and Theories. 1959.

MOUZON, EDWIN DUBOSE AND REES, PAUL K. Mathematics of Finance. 1959.

TOU, TSU-LIEH. Digital and Sampled-Data Control Systems. 1959.

METALLURGY

HARRINGTON, CHARLES D. AND RUEHLE, ARCHIE E., ED. Uranium Production Technology. 1959.

MATHEWSON, CHAMPION HERBERT. Zinc; the Science and Technology of the Metal, its Alloys and Compounds. 1959.

PETROLEUM TECHNOLOGY

NOEL, HENRY MARTYN. Petroleum Refinery Manual. 1959.

PHYSICS

Advances in Semi-Conductor Science. 1959.

ELTON, L. R. B. Introductory Nuclear Theory. 1959.

Progress in Cryogenics. Vol. 1. 1959.

SYMPOSIUM ON FRICTION AND WEAR. Friction and Wear; Proceedings . . . Detroit, 1957. 1959.

PLASTICS

SIMONDS, HERBERT RUMSEY. Source Book of New Plastics. 1959.

ZADE, HANS PETER. Heatsealing and High-Frequency Welding of Plastics. 1959.

FRANKLIN INSTITUTE LABORATORIES
NICOL H. SMITH, DIRECTOR

PRODUCING BURNS FOR MEDICAL RESEARCH

BY

C. W. HARGENS[1]

Progress in the study of burns will be advanced by the new radiant heat source shown in Fig. 1. Medical research workers can, through its use, make more discriminating evaluations of healing techniques because experimental burns can be produced under repeatable conditions. The equipment developed at The Franklin Institute for Drs. Irving Sokolic and Joseph Foreman at the Einstein Medical Center of Philadelphia is being used in the hospital's burn research program. The instrument, which was designed to produce standardized, second- and third-degree burns, presented numerous interesting problems during its development.

Burns are usually caused by contact with hot objects, or they may be produced by radiant energy, as from the heat of nuclear explosions. The new instrument produces burns of the second type by means of an

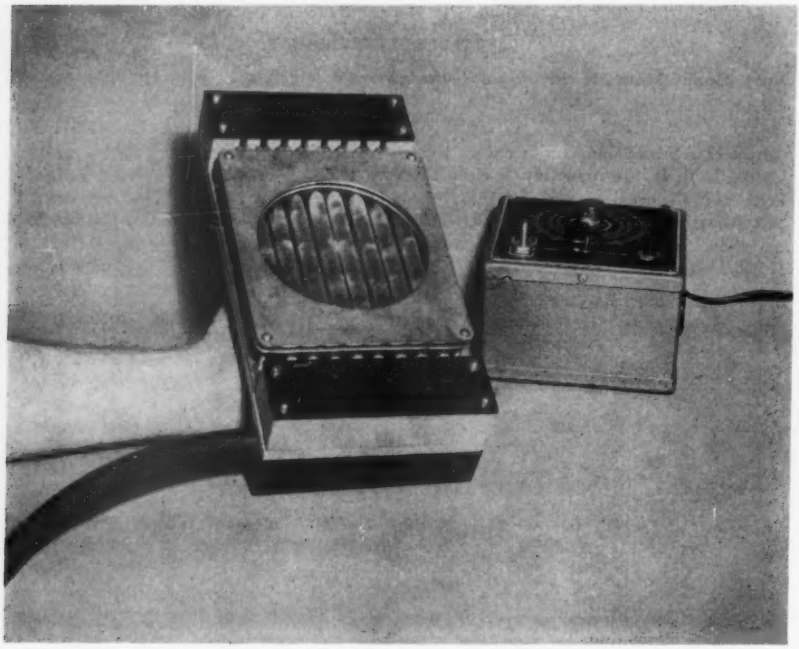


FIG. 1.

[1] Head of The Franklin Institute Laboratories' Bioelectronics Branch.

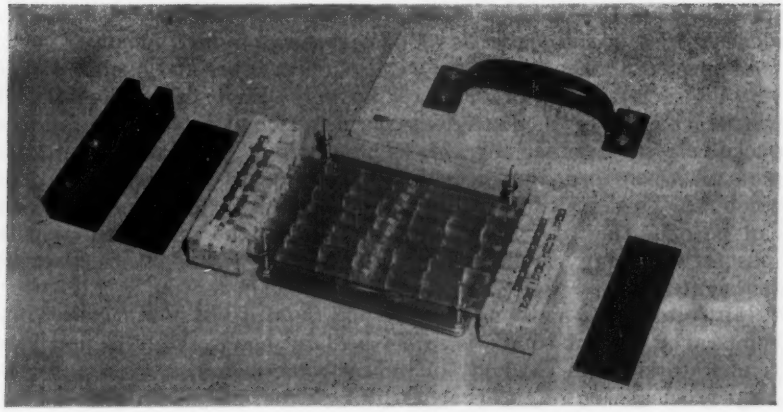


FIG. 2.

intense radiant flux. This flux is controlled so that a known amount of energy falls upon the skin of the subject. In the present design roughly 4 kilowatts of electric power flow from the 230-volt mains, and most of this is passed on to the burn areas. These range in size, by means of an adjustable aperture and variable exposure time, from second-degree burns 4 in. in diameter to third-degree burns 1 in. in diameter.

It is important in such a vigorous process to use high energy levels and keep the exposure time as short as possible. This is necessary because the person receiving the burn can control his survival urge for only a brief moment. The involuntary, protective mechanisms of his body then take over and cause him to pull away from any source of severe pain.

Moreover, there are biological differences between a burn caused by an intense flash of radiant energy and one resulting from a slow broiling of the skin layers. This development was intended to simulate the former. In the event of a nuclear war, burns of this type could present an enormous problem.

The design of the instrument was based upon a number of practical requirements including portability. Experiments with it are usually conducted in penal institutions, where volunteers are readily willing and receive attractive compensation. It is important under the circumstances to avoid the use of heavy laboratory apparatus or unusual fuel sources to produce the high temperature.

A further consideration in the design of a workable device was the problem of disposing of huge amounts of heat. This led to an early decision to find a system which could be turned on only during the exposure period or just slightly longer. Continuously burning furnaces

seemed out of the question on the basis of heat calculations, and sun-focusing was too unreliable as well as expensive to instrument.

Tungsten filaments enclosed within quartz tubes and operated at a very high temperature provided the answer. Eight tubes rated at 500 watts each produced a tremendous concentration of radiant energy when packed closely together (Fig. 2). These units also rise rapidly in temperature when turned on, reaching 80 per cent of peak output in two seconds. This is possible owing to the low mass of the filaments, which can be operating at more than 3000 F; also, the quartz tubes pass most of the energy immediately without being raised to the filament temperature.

In spite of the high temperature and quantity of power being consumed, the radiant source is safely held by hand. Only a moderate temperature rise in the body of the instrument results, even during a lengthy succession of experiments. The structural materials used are mostly stainless steel and a cement-asbestos composition. Hundreds of quantitatively administered burns are now under study. In addition to the treatment data the results give relative response of variously pigmented skins to intense radiation. Reports of the specific details will appear in medical journals.

BOOK REVIEWS

PROGRESS IN CRYOGENICS, Vol. 1, edited by K. Mendelssohn, 259 pages, plates, diagrams, 6 × 9¾ in. New York, Academic Press, Inc., 1959. Price, $11.00.

In the preface to this first volume, Dr. Mendelssohn describes the objective of the new series as the provision of "summarizing articles on the whole field of low temperature methods, as distinguished from low-temperature physics and chemistry." Since the rapid increase in the utilization of low temperature phenomena appears to have outstripped the literature on the subject, this new series should have particular value in providing authoritative information on current activity in a wide variety of subjects. In view of the broad objectives of the series, and the international distribution of the authors, it is not surprising to find some variation in the form of contributions. Several of the articles are relatively brief while others take the form of fairly extensive reviews, particularly of topics in which there has been much activity within recent years.

In "Superconducting Circuits," Dr. D. R. Young provides a very brief description of the phenomena of superconductivity and a more lengthy discussion of the various circuit elements which involve superconductivity in their operation. A fairly detailed analysis of several basic circuits is followed by a largely qualitative discussion of superconducting systems, with principal emphasis on those applicable to computers. The author concludes with some remarks on the direction in which future development must proceed if the feasibility of high capacity computers, using superconducting circuit elements, is to be demonstrated.

"Thermoelectric Cooling" by Dr. D. A. Wright is a comprehensive review of the subject. Approximately half of the chapter is devoted to the thermodynamics and theory of semiconductors, basic to the subject. This is followed by a detailed discussion of the properties of relevant materials, with particular emphasis on bismuth telluride. In the final section Dr. Wright analyzes the performance of multiple refrigeration units arranged in cascade, makes a comparison with the demonstrated performance of the compressor type of refrigerator and concludes with a brief description of various devices using thermoelectric cooling.

A short historical summary precedes the review of present information in the chapter "Evacuated Powder Insulation for Low Temperatures" by M. M. Fulk. Considerable progress has been made in the development of techniques of thermal insulation which are of particular interest to the engineer concerned with the storage and transportation of large quantities of liquid refrigerants. In this connection, the compilation of the measured thermal conductivities of many different insulating materials is particularly valuable.

A significant portion of "Distillation at Low Temperatures" by B. R. Brown is devoted to a critical review of the progress which has been made towards understanding the detailed processes involved in the fractional distillation of liquid mixtures. In the description of particular systems, emphasis is laid on the distillation of liquid air and of liquid hydrogen, although mention is also made of the separation of boron, carbon and helium isotopes. In conclusion, the author discusses areas in which further fundamental investigation must be performed if the efficiency of distillation processes is to be improved.

In "The Measurement of Mechanical Properties of Metals at Low Temperatures" Dr. H. M. Rosenberg describes in some detail the experimental techniques which have been used to investigate the low temperature behavior of the elastic moduli, creep, fatigue and internal friction of metals. Discussion of the experimental results is brief but is adequate to justify the author's contention that this branch of low temperature physics can aid considerably in the study of metals.

Dr. G. H. Minkoff presents an extensive review in the chapter "Frozen Free Radicals." There is a brief historical introduction followed by a review of experimental methods.

The information obtained from infra-red, optical, ultra-violet and paramagnetic resonance absorption investigations of trapped free radicals is discussed in detail and the broad scope of the results which may be obtained from this area of research is emphasized. An extensive list of references is provided.

The two chapters, "Low Temperature Calorimetry" by Dr. R. W. Hill and "The Determination of Specific Heats by the Temperature-Wave Method" by N. V. Zavaritsky, are complementary and, at the same time, provide an interesting comparison. The former chapter consists of an extensive review of the techniques of calorimetry and thermometry at sub-ambient temperatures, particular attention being paid to the range of temperatures below 4 K. Below 1 K, however, the conventional method of specific heat determination involving the measurement of a temperature change produced by addition of a known amount of energy to the sample, runs into increasing experimental difficulties. It is in this range that the temperature-wave technique becomes valuable. Originally proposed by Angstrom, the method has only been used in low temperature research within recent years. With further refinement of the experimental techniques, which are described in considerable detail in the second chapter, it is reasonable to expect an increasing use of this elegant method in the temperature range below 1 K.

"Ultrasonic Attenuation in Metals at Low Temperatures" by Professor R. W. Morse is valuable as a review of the contributions made to the knowledge of electron-phonon interactions in metals by ultrasonic attenuation experiments. Results obtained both in normal and in superconducting metals are discussed, with, in the latter case, particular attention being given to a comparison of the experimental observations with the predictions of the Bardeen, Cooper and Schrieffer theory.

Dr. Mendelssohn and his contributors have maintained a high standard of presentation throughout and have produced a volume which will be useful to the specialist in low temperature physics and chemistry, to the engineer and to the advanced student making his first acquaintance with the rapidly expanding field of cryogenics.

JAMES NICOL
Arthur D. Little, Inc.

HYPERSTATIC STRUCTURES, by J. A. L. Matheson. 474 pages, diagrams, $6 \times 9\frac{1}{4}$ in. New York, Academic Press, Inc., 1959. Price, $15.50.

In a day when most textbooks are written mainly to satisfy administration demands for publication, it is natural that few show the careful preparation and genuine concern for the student required for success. Therefore when a book does come along which appears to have been motivated primarily by a real desire to teach something, it deserves to be noticed. A textbook, after all, is supposed to help a student learn a subject he does not already know. To do this successfully it must be informed by all those same principles which make for good classroom teaching: organization which allows the subject to develop in a logical and natural fashion; emphasis on unity of ideas instead of diversity; and an unflagging effort toward clarity. On all these counts, *Hyperstatic Structures* is a distinctly superior text.

Although only one has appeared, there are to be two volumes in the set. All those elaborate and detailed problems with which the engineering teacher simulates practice are relegated to volume two. The present volume is entirely devoted to the exposition of basic ideas and theory. The happy result is that it can be read through from first to last without losing the thread of the argument or bogging down in pages of calculations. This is not to say there are no sample problems: nearly every page contains some simple sketch, illustration or example which serves to reinforce the remarks in the text, but which does not halt or confuse the reader. One can simply glance at them, get the point immediately, and pass on. The book is not a novel, of course, but devices such as these, coupled with an easy and lucid style, contribute to readability and clarity far above the average.

Another strong point is the emphasis placed on the underlying unity of apparently different techniques of analysis. One of the common failings of texts and courses in indeterminate structures is the impression often given the student that the many analytical routines are all in fact quite different "methods" with fully independent status. Beginning in the preface, where a "family-tree" diagram appears, the author goes to great lengths to

counter this impression, taking every opportunity throughout the text to point out common features and lines of descent. Except for final chapters on elastic stability and matrix analysis, there are few topics in this book which do not appear in most others on the same subject. The way this material is put together and presented, however, makes it a more useful text than most. It should be especially welcomed by the student who must learn alone, and who otherwise might find such unifying insights difficult to acquire.

WILLIAM C. YAGER
The Franklin Institute Laboratories

CORROSION OF CHEMICAL APPARATUS, by G. L. Shvartz and M. M. Kristal. Translated from the Russian, 250 pages, illustrations, 6 × 9 in. New York, Consultants Bureau, Inc.,; London, Chapman & Hall, Ltd.; 1959. Price, $7.50.

This is not another book on the corrosion of steel in the conventional manner. Nor is it a general treatment on the corrosion of metals. Rather, it is a monograph on one phase of metallic corrosion—intercrystalline corrosion and corrosion cracking.

In corrosion testing we are prone to determine the over-all weight loss of a specimen in a given medium, calculate the mean penetration per unit time, and use this calculated value of the penetration to determine the safe life of a piece of equipment. The whole object of this book is to show that this procedure is unsafe—that the over-all weight loss of metal in a given environment can be very small, but the metal can fail in the environment—and that quite suddenly, at times.

The approach of the book to the corrosion cracking of metals is practical in the extreme. It is divided into six chapters: the first covers the general causes of corrosion cracking in metals; the second, third, and fourth, cover corrosion cracking in carbon steels, stainless steels, and non-ferrous alloys, respectively; the fifth covers methods of protection and prevention; and the sixth chapter covers testing methods.

Under the causes of corrosion cracking the authors discuss metal structure, industrial treatments of metals, surface treatments, the effect of environments, stresses and their effect, and cracking mechanism.

These various physical and chemical variables are then discussed with respect to the various metals and alloys. There is a large amount of useful information in Chapters 2, 3, and 4, that would seem to be of use to manufacturers of metals, fabricators, designers of metal products, and to the user.

The discussion of methods of test was of particular interest to the reviewer. A large amount of research is being carried out in the U. S. A. on stressed metals in corrosive environments. Apparatus and techniques for these studies are well presented in Chapter 6.

The authors' approach to the subject seems to be that of the metallurgist and operating engineer, rather than the chemist. Many experimental data are given, usually tabulated for reference. Unfortunately there is no index, although the table of contents is quite useful. A bibliography of 219 references, of which many are to non-Russian sources, is attached.

Corrosion of Chemical Apparatus should be well received in its English translation.

GEORGE S. GARDNER
Amchem Products, Inc.

THE TRANSITS OF VENUS, by Harry Woolf. 258 pages, illustrations, 6 × 9¼ in. Princeton, Princeton University Press, 1959. Price, $6.00.

Once in a blue moon there appears a definitive work on an astronomical subject written by a skillful writer, scholar and historian. Such a book is *The Transits of Venus* by Harry Woolf, associate professor of history at the University of Washington.

In his preface the author states: "The usefulness of the transits of Venus in determining the fundamental unit of astronomical measurement, the solar parallax, or the mean distance of the earth from the sun, was first demonstrated by Edmund Halley at the turn of the 18th century. . . ." Thus, the problem was stated.

The cold logic of Halley appealed to the other scientists who modified his methods and finally led expeditions all over the world.

In order to observe the transits of 1761 and 1769 the cooperation of the learned societies of that day was enlisted. Observers all over the world—in Peking, Calcutta, Spain, Russia, Siberia, Sweden and America—observed the

ingress and egress time. From the results of these observations came the first realistic solar parallax.

A staggering volume of research by the author went into this work. The author says: "The archival records of the expeditions, the private correspondences between scientists, and the unpublished memoirs of learned societies" were consulted. In many cases to attain his objective the author includes the source material. The book rings with authority.

While essentially this book is the record of the preparations of the various expeditions for the two transits the author weaves a fascinating story in which are included the adventures of the various groups. The historian's touch, a most pleasant treatment in this type of book, is apparent throughout.

Professor Woolf is to be congratulated on authoring a classic treatment on the derivation of one of the fundamental units in astronomy. His work will appeal to both the historian and the astronomer.

I. M. LEVITT
The Fels Planetarium

ZINC, edited by C. H. Mathewson. ACS Monograph No. 142. 721 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1959. Price, $19.50.

This monograph on zinc, one in a series of scientific and technologic monographs produced and published by the American Chemical Society, has been prepared with the cooperation of the American Zinc Institute. The book consists of fourteen chapters and represents the contributions of over fifty authors.

The first three chapters very briefly cover the historical background, the economics and statistics of the zinc industry and the geology of zinc deposits.

Chapter 4, the most theoretical in the book, deals with the physical chemistry of specific reactions associated with the roasting of zinc sulfide and with the methane and carbon reduction of zinc oxide. Standard free energies of various zinc reactions from K. K. Kelly, Rossini and others are presented in tabular form. Other data including the electronic arrangement of zinc atoms, crystal structure of the metal zinc and some of its compounds, and numerous other physical constants are presented.

Chapters 5 through 9, entitled The Treatment of Ore or Concentrate for Subsequent Processing by Leaching or Pyrometallurgical Methods, Metallurgical Extraction, The Refining of Zinc, The Manufacture of Zinc Oxide, and The Processing and Uses of Metallic Zinc and Zinc Base Alloys constitute approximately 70 per cent of the entire text. These chapters are lengthy and highly descriptive and deal for the most part with industrial processing. Equipment, special practices and techniques presently in operation in the manufacture and uses of zinc in this country and abroad are painstakingly described. A brief discussion of the physical metallurgy of zinc is also given in one section of Chapter 9 in which some metallographic techniques and micrographs of zinc and zinc base alloys are presented. Special attention is given to casting and die casting techniques, galvanizing and other galvanic protective applications of zinc.

Chapter 10 covers zinc as an alloy agent and presents many binary and ternary equilibrium diagrams.

The final chapters cover industrial uses of zinc oxide, zinc sulfide and other zinc compounds, the physiology of zinc and its importance to human life, and zinc as a wood preservative and its uses in agriculture.

The text is somewhat disjointed but provides in a single volume many facts and figures about zinc and the zinc industry. It should serve as an excellent reference for student, researcher and industrial engineer alike.

VICTOR DAMIANO
The Franklin Institute Laboratories

ADVANCED MECHANICS OF FLUIDS, edited by Hunter Rouse. 444 pages, diagrams, 5¾ × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $9.75.

Nominally a sequel to Hunter Rouse's well-known *Elementary Mechanics of Fluids*, this book is basically different in several respects, of which multiple authorship is the least important. In the earlier work the emphasis was on physical phenomena. In this one the emphasis is on analytical techniques and philosophy. In fact, the stated aim of the book is to equip advanced students and workers with the theoretical and analytical tools to advance the frontiers of fluid mechanical knowledge.

Approximately the first fifth of the book is

given over to discussions of basic concepts, fundamental equations, and dimensional considerations. Six subsequent chapters cover irrotational flow, conformal mapping, laminar flow, boundary layers, and free-turbulent shear flow, including mixing.

The author remarks that the reader's preparation is assumed to include only a year of advanced calculus. It would appear, however, that a good course in complex variables would be a help, since the treatment in the text seems too brief to be self-contained. One notes that, following engineering rather than physical tradition, vector methods are not used.

The formal apparatus of a text is present—examples, problems, answers etc.—making the book suitable for use in the graduate classroom. But this excellent work will probably find its way most naturally to the reference shelf in the working laboratory.

WILLIAM C. YAGER
The Franklin Institute Laboratories

ADVANCES IN ASTRONAUTICAL SCIENCES, Vol. 4, edited by Horace Jacobs. 460 pages, illustrations, $6\frac{1}{2} \times 9\frac{3}{4}$ in. New York, Plenum Press, Inc., 1959. Price, $8.00.

The American Astronautical Society has earned an enviable reputation as an organization dedicated to space flight. While the Big Brother organization—The American Rocket Society—has long held a monopoly on this field the A. A. S. by concentrating on space flight mechanics to the exclusion of hardware, in the rocket engine sense, provides the ideal platform for serious astronauts.

Its annual meetings are well attended and the caliber of the papers given is high. It has been the custom to collect these papers to be published as the proceedings of the annual meeting.

Advances in Astronautical Sciences is Volume 4 of this series. It is an outstanding publication containing significant papers by ranking figures in astronautics. Men such as Fleisig, Van Allen, Singer, Petersen, Hoover, Jacchia and Pickering have authored papers in this volume.

The editor has divided astronautics into six sections dealing with Upper Atmosphere Research and Re-entry Mechanics, Space Vehicle Design, Guidance and Instrumentation, Satellite Mechanics and Space Exploration, Rockets and Satellites and Man's Environment in Space.

It is difficult to review a book of this type in which there are some 35 papers on as many topics. There are several outstanding ones to this reviewer, and at this point he would like to emphasize that this is purely a subjective selection with which few may agree. I have chosen the following six as significant: 1. Horizon Scanning for Atmospheric Re-entry, by E. J. McCartney; 2. On Gravitation, by Vjekoslav Gradecak; 3. Assembly of a Multi-Manned Satellite, by Richard A. Byers and Saunders B. Kramer; 4. Perturbed Motion of Selenoid Satellites, by E. T. Benedikt and Paula S. Levin-Cohen; 5. Artificial Modifications of the Earth's Radiation Belt, by S. Fred Singer; and finally, 6. George W. Hoover's paper on The Man-Machine System in Space Vehicles.

Lest the reader think this is just another book on astronautics which can be read at a sitting, this reviewer emphasizes that the papers contained in this volume are of the highest research calibre. They represent the result of long researches and are fully developed so that unless one has a proper mathematical background the papers are quite likely to appear unintelligible. But to the serious worker in the field who not only is looking for results to guide him in his own researches, but looks for the development of those results this is an ideal book.

In all probability, this book will not be a "best seller," but it can be a welcome addition to the worker in the field. The A. A. S. is to be congratulated in assembling this talent and making the proceedings available to all.

I. M. LEVITT
The Fels Planetarium

BOOK NOTES

PRINCIPLES OF OPTICS, by Max Born and Emil Wolf. 803 pages, illustrations, 6½ × 9¾ in. New York, London, Paris, Los Angeles, Pergamon Press, 1959. Price, $17.50.

This new work will be welcomed by those who have for 25 years depended upon Max Born's *Optik*. The present work is not a translation of the German book; in fact, it omits a great deal of the *Optik* material to make room for the important work in optics which has been done since 1933. The authors cover only the optical phenomena which may be treated in terms of Maxwell's phenomenological theory, thus eliminating most connections with atomic physics, quantum mechanics and physiology. Fourteen chapters deal with such phases of optics as: properties of the electromagnetic field (scalar, vector, and plane waves); geometrical theories of optical imaging (stigmatic, astigmatic, chromatic, etc.) and aberrations (primary and chromatic); instruments (camera, telescope, microscope); theories of interference and interferometers; diffraction (both with partially coherent light, and rigorous diffraction); diffraction by ultrasonic waves; and the optics of metals and crystals. This is a detailed piece of work, made even more valuable by the inclusion of nine appendices on mathematidal techniques such as the calculus of variations, the Dirac delta function, etc.

ANNALS OF THE INTERNATIONAL GEOPHYSICAL YEAR. VOLUME I, 446 pages, plates, 7½ × 9¾ in. London, New York, and Paris, Pergamon Press, 1959. Price, $17.00.

Pergamon Press has undertaken to publish a series of volumes encompassing results obtained during the recent International Geophysical Year. Published at irregular intervals for the International Council of Science Union's Special Committee for the IGY, the volumes will record data and observations as well as authors' conclusions drawn from the data. Contributions may be submitted either in English or French; in the printed volumes, each paper (if we are to judge succeeding ones by Volume I) will appear in both languages.

Volume I serves as the introduction to the series, for it contains three papers covering in detail the First International Polar Year, the Second International Polar Year, and the Inception and Development of the International Geophysical Year. Sydney Chapman, President of the Special Committee, has a short introduction to the volume. The two papers on the International Polar Years represent contributions from several authors; the final paper on the development of the IGY is by Sir Harold Spencer Jones. We look forward to the succeeding volumes for a concise and accurate report of the accomplishments during the IGY.

ELECTRON PHYSICS, by O. Klemperer. 248 pages, illustrations, 5½ × 8½ in. New York, Academic Press, Inc.; London, Butterworths Scientific Publications; 1959. Price, $7.00.

Useful primarily as a text for undergraduates, *Electron Physics* (whose subtitle is *The Physics of the Free Electron*) includes the subjects of electron dynamics and electron optics. The author divides his text into two parts of five chapters each, and adds seven appendices. In Part I, which leans more to the practical side, the material covers electron motion, electron optics, electronic space charge, and the detection of cathode rays. Equipment and methods are described. In Part II the approach is more fundamental, dealing with various properties of the electron such as charge, mass, wave-length, spin and magnetic moment. Part I will appeal more to engineers, while Part II will appeal more to physicists. References and problems follow each chapter.

MATHEMATICAL ANALYSIS, by Edwin M. Hemmerling. 332 pages, diagrams, 8¼ × 5½ in. New York, McGraw-Hill Book Co. Inc., 1959. Price, $5.95.

This text represents an ideal one-semester college course. Providing a survey of the essential knowledge and skills necessary for successful study of calculus, the material pre-

sented begins with fundamental algebraic equations and goes through elements of differential calculus. Attention is particularly given to those topics and concepts which technical and pre-engineering students must thoroughly understand before they tackle the study of calculus. Special emphasis is given to analytic relationship between trigonometric functions rather than on the solution of triangles. Extensive treatment has also been given to Radian measure.

FOUNDRY ENGINEERING, by Howard F. Taylor, Merton C. Flemings and John Wulff. 407 pages, diagrams, $8\frac{1}{4} \times 5\frac{1}{2}$ in. New York, John Wiley & Sons, Inc., 1959. Price, $9.75.

This volume deals with the ancient art of metal casting, born in the Bronze Age and passed down through the centuries in a traditional aura of secrecy. The authors have emphasized the fundamental concepts that apply to all cast metals. They have presented a comprehensive, scientifically correct approach to engineering principles and methods of foundry operation. Chapters deal in solidification and structure of cast metals and in the principles of correct gating and risering of castings. Pertinent topics are covered, such as refractories, casting processes, melting equipment and practices, sand reclamation and control, welding and heat treatment. Written in layman's language, the practical man as well as the college engineering student, will find this concise text valuable.

FLAT ROLLED PRODUCTS: ROLLING AND TREATMENT, edited by T. E. Dancy and E. L. Robinson. 128 pages, diagrams, 6×9 in. New York and London, Interscience Publishers, 1959. Price, $3.75.

This is Volume 1 of a new series of Proceedings of Technical Conferences Sponsored by The Metallurgical Society of the American Institute of Mining, Metallurgical and Petroleum Engineers. Held in Chicago in January, 1959, the Conference presented six papers with discussions following some. Two of these papers (and a panel discussion) dealt with factors affecting the shape and surface texture of hot and cold rolled products; four covered treating of rolled products.

INDUSTRIAL THERMAL INSULATION, by A. C. Wilson. 295 pages, illustrations, 6×9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $9.00.

This is the first text devoted exclusively to the problems of insulating and, since it is a practical book, it will be useful not only for students but also for those engaged in the manufacture or use of insulating materials. The text includes old and new insulating materials, methods of and materials for applying insulation to hot surfaces, materials and methods used in low-temperature insulation, and the principles involved in insulation. Several useful tables (for example, recommended storage temperatures for various foodstuffs) are given. The new materials covered are fibrous and cellular glass products, hydrous calcium silicate, and foamed plastics.

FERRITES, by J. Smit and H. P. J. Wijn. 369 pages, diagrams, 6×9 in. New York, John Wiley & Sons, Inc., 1959. Price, $10.00.

Many actual problems of the theory of phenomena occurring in ferromagnetic materials during various processes of magnetization are dealt with in this text. Greater use is made of simple physical models than of rigorous mathematical derivations. The characteristics of ferrites that are of importance for application purposes are presented and discussed and, whenever possible, explained in terms of intrinsic properties. The authors have treated the subject of ferrites on an intermediate level. The book is intended for all those actively interested in the properties of ferromagnetic oxides.

MODERN ELECTRONIC COMPONENTS, by G. W. A. Dummer. 472 pages, diagrams, $5\frac{1}{4} \times 8\frac{1}{2}$ in. New York, Philosophical Library, Inc., 1959. Price, $15.00.

This book represents a comprehensive survey of the characteristics of the more common components, together with information on their behavior under the arduous environmental conditions to which they are now frequently subjected. The author has included chapters on component specifications, transistor-circuit components, reliability, and future development in the field, as well as selected bibliographies. In choosing a suitable

component for a particular purpose, this volume should be of value to many engineers concerned with the design, development, production and use of electronic equipments.

PRECISION VALLEY: THE MACHINE TOOL COMPANIES OF SPRINGFIELD, VERMONT, by Wayne G. Broehl. 274 pages, plates, 6 × 9 in. New York, Prentice-Hall, Inc., 1959. Price, $5.95.

The story of three key firms in the machine-tool industry in Springfield, Vermont, spans a seventy-year period. Through the corporate lives of these companies can be seen the difficult management decisions demanded, the fantastic war-time buildups, the deep cyclical swings characteristic of the industry, and the continuing need for innovation in the face of widely fluctuating profits. Discussed in detail are the heavy machine-tool purchases made by the Russian government in the early 1930's and later in the same decade, important sales to Japan. The three companies in meeting all the issues inherent in the national machine-tool picture also adapted to the peculiar problems dictated by their Vermont environment. A uniquely New England flavor is given the story.

PUBLICATIONS RECEIVED

PRODUCT ENGINEERING DESIGN MANUAL, edited by Douglas Greenwood. 342 pages, diagrams, 8½ × 11 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $9.00.

CHEMICAL PROCESS PRINCIPLES. PART TWO: THERMODYNAMICS, by Olaf A. Hougen, Kenneth M. Watson and Roland A. Ragatz. Second edition, 624 pages, 5¾ × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $9.75.

CALCULUS WITH ANALYTIC GEOMETRY, by Richard E. Johnson and Fred L. Kiokemeister. Second edition, 709 pages, diagrams, 6 × 9 in. Boston, Allyn and Bacon, Inc., 1960. Price, $8.75.

DICTIONARY OF DISCOVERIES, by I. A. Langnas. 201 pages, 6 × 9 in. New York, Philosophical Library, 1959. Price, $5.00.

A HISTORY OF SCIENCE, TECHNOLOGY & PHILOSOPHY IN THE 16TH & 17TH CENTURIES, by A. Wolf. Reprinted from the second edition. Vol. I, 349 pages, plates, 5¼ × 8 in.; Vol. II, 337 pages, plates, 5¼ × 8 in. New York, Harper & Brothers, 1959. Price, $1.95. each (paper).

SPACE, TIME AND GRAVITATION, by Sir Arthur Eddington. Reprint of the original 1920 edition, 216 pages, 5¼ × 8 in. New York, Harper & Brothers, 1959. Price, $1.35 (paper).

READINGS IN THE LITERATURE OF SCIENCE, edited by William C. Dampier and Margaret Dampier. Reprinted edition, 275 pages, diagrams, 5¼ × 8 in. New York, Harper & Brothers, 1959. Price, $1.50 (paper).

ON UNDERSTANDING PHYSICS, by W. H. Watson. Reprinted edition, 146 pages, 5¼ × 8 in. New York, Harper & Brothers, 1959. Price, $1.25 (paper).

INTRODUCTION TO MATHEMATICAL THINKING, by Friedrich Waismann. Translated and reprinted from the 1951 edition, 260 pages, diagrams, 5¼ × 8 in. New York, Harper & Brothers, 1959. Price, $1.40 (paper).

SOIL, GRASS AND CANCER, by André Voisin. 302 pages, plate, 5½ × 8½ in. New York, Philosophical Library, 1959. Price, $15.00.

CURRENT TOPICS

Yttrium as an Alloying Metal.—Spectacular improvements in steel alloys made by adding yttrium, a rare metal, may hold the key to heat-resistant metals needed in atomic flight and the space age. Scientists from the General Electric Aircraft Nuclear Propulsion Department (ANPD) told an American Society for Metals—Atomic Energy Commission symposium that yttrium and in some cases other rare metals have an extremely beneficial effect on "workability, grain refinement and resistance to recrystallization at elevated temperatures" of various steel alloys.

Most extraordinary effect of the addition of yttrium, they said, is the improvement in high-temperature oxidation resistance of iron-base alloys containing chromium.

An outstanding example, they pointed out, is AISI-type 446 stainless steel, containing 25 per cent chromium, with the balance primarily iron, which is oxidation-resistant to 2000 F. By alloying a small amount of yttrium to this composition, the scientists reported, "an alloy is obtained whose oxidation resistance is as good at 2500 F. as is the alloy not containing yttrium at 2000 F."

They said that "the addition of 1 per cent yttrium is as effective as 5 per cent aluminum in improving the oxidation resistance of the iron-chromium-base metals."

The iron-chromium-yttrium alloy, they found, "is readily cold-rolled, easily welded and represents a significant development in high-temperature metallurgy."

Satisfactory alloys which will withstand high temperatures are among the most important requirements for atomic flight and the space age.

A variation of this alloy, which provides even greater oxidation resistance, the General Electric team reported, includes a fourth element, either aluminum or thorium.

"The combination of iron-25 chromium-yttrium with 3 per cent aluminum provides an oxide coating at temperatures of 2600 F. which is enamel-like, hard, non-chipping and thermally shock resistant," the scientists said.

They pointed out, however, that applications for this alloy may be limited by its fabrication problems, poor weldability and difficulty in brazing, plus its embrittlement after lengthy heating.

Nonetheless, they pointed out, it provides a wrought alloy "unparalled in high-temperature metallurgy."

The ANPD-led studies revealed that yttrium and some rare earth elements are effective in improving the barrier properties of chromium oxide to nitrogen and oxygen at temperatures up to 2500 F. By blocking entry of nitrogen and oxygen, yttrium improves the desirable properties of chromium alloys. Such properties include ductility and fine grain structure.

From the ductility standpoint, yttrium added to vanadium is a spectacular performer, the ANPD report stated.

Studies revealed that yttrium removes oxygen and nitrogen from vanadium to improve ductility.

The scientists reported that in tests, unalloyed vanadium cracked exten-

sively when cold-pressed and cold-rolled. After the addition of 0.5 to 2 per cent of yttrium "cold-rolling to thin strip was accomplished with only slight edge roughness," they said.

The scientists said that many applications of yttrium and rare earth elements to metal alloys show great potentialities, depending only on the ability of industry to translate laboratory results to useful high-volume product applications.

They said that part of the work reported was carried out by the Battelle Memorial Institute at Columbus, Ohio, and by Denver Research Institute, Denver, Colo.

Photo-Optical Memory System.—Photographic plates used in the permanent memory of an experimental electronic telephone switching system are now being exposed and developed automatically, according to Bell Telephone Laboratories. The permanent memory stores such things as directory information and instructions for the electronic switching system. This information is in the form of thousands of tiny clear dots on an otherwise opaque photographic area 2 in. square. Seventeen areas are used together to make up a complete photographic plate, and the entire permanent memory store is comprised of four such plates.

The spots of clear and opaque film are scanned by a beam which is generated by a cathode ray tube and focused by a system of lenses. This operation gives rise to the name of the unit—"Flying Spot Store." The movement of the beam is controlled by an electronic and optical positioning system, which must be accurate enough to pick out on each area a particular spot location.

The flying spot store contains over two million bits of information as opaque or transparent spots, and can read out a 68 bit word (that is, 68 spots simultaneously) every two and a half microseconds. Random access to any word adds a major feature of versatility.

The store itself is used to expose the plates. The moving beam from the cathode ray tube is focused through an objective lens onto the photosensitive area. The spot is moved over the entire 2-in. square image area and stopped briefly at positions where a spot exposure is desired. A photographic emulsion was selected which is "slow" enough not to be exposed by the passing of the beam alone, but requiring the lengthened exposure of the pause to make a record.

Each of the seventeen small areas on a plate is exposed individually, in sequence, while the others are covered by a system of shutters. Automatic control of this operation allows an entire plate, containing 550,000 spot positions, to be exposed in less than three minutes.

After exposure, the plates are withdrawn from the memory unit in a dust- and light-proof container and inserted into the automatic processor for developing and fixing. Two of the plates can be processed simultaneously.

Once the plates are inserted in the processor, they are automatically carried through a number of steps. They are dipped into one tank after another, depending on a predetermined schedule. As many as fifteen steps may be involved in a typical reversal processing schedule, and appropriate dwell times are also set to allow proper processing.

After developing, reversal, and fixing operations have been completed, the plates are rinsed and dried in filtered air, and delivered back into their container for reinsertion into the memory unit.

The time required for a complete developing schedule is less than a half hour. Once the machine has been put in operation, it will continue through its sequence without interruption. The machine has been made simple to operate since it is expected that personnel unfamiliar with photographic procedures will operate it.

Airborne Crane-Shovel.—An airborne crane that can be air delivered by parachute in forward areas has been developed through a joint effort of the U. S. Army Engineer Research and Development Laboratories, Fort Belvoir, Va. and the Crane-Shovel Industry, to meet the need for lightweight, yet rugged construction equipment.

The crane not only is air transportable and air droppable, but is unique in that it is capable of lifting more than its own weight. This newest item of airborne equipment weighs only 16,000 lb., yet it has lifted more than 18,000 lb. in various tests. It tows readily at convoy speeds, has a self-propelled speed of 8 mph., and can be converted to $\frac{3}{8}$ cu. yd. clamshell, dragline, and shovel front operation.

The crane is equipped with a folding gantry and three-piece boom. In preparation for air drop the gantry is folded so that it is below the machinery housing and the boom removed. The boom sections are light enough for handling by two men without the use of auxiliary lifting equipment. When the boom is removed the total length of the machine is 17 ft.

To reduce weight aluminum has been used for the crane cab, gear cases, outrigger beams, and outrigger pads.

Letter-Sorting Machine.—The world's largest capacity letter sorting machine—a completely "made in America" giant, capable of sorting 43,000 letters an hour to 279 destinations was demonstrated recently in Detroit at the Armory Building adjacent to the Roosevelt Park Annex Post Office. Demonstrations were presided over by Postmaster General Arthur E. Summerfield and Ray R. Eppert, President, Burroughs Corporation. Burroughs built the device, the first of ten being built by the Detroit business machine and electronics firm under a $1,869,000 research and development contract with the Post Office Department.

While more than doubling the sorting capacity of any existing machine, the new machine is only slightly larger (78 ft. long and 10 ft. high) than an imported sorting machine installed in the post office at Silver Spring, Maryland, and another model installed in the Washington, D. C. post office, developed by foreign interests and used for experimental purposes.

"Machine letter sorting makes drastic improvements in the way the Postal Service handles letters," Mr. Summerfield said. "Since the days of Benjamin Franklin," he explained, "a tedious 'peek and poke' method of sorting letters into pigeonholes has been used."

With machine sorting, letters move automatically in front of a human operator seated at a keyboard. As each letter passes, the operator presses several keys. This sets wheels in motion inside the machine to route the letter to any one of 279 destination slots.

In addition to having the greatest capacity and being the first made entirely in the United States, the new Burroughs Letter Sorter is the first to be readily adaptable for use in post offices of varying sizes, Mr. Summerfield pointed out.

A feature of the new machine is that a single sorting machine can be equipped with varying numbers of standardized identical ("modular") units and supported by input stations

from one to a total of twelve keyboards. Detroit, the sixth largest post office in the United States, will have the maximum 12 sorting keyboards but a medium size post office could have, for example, 6 of the consoles to start with, and other keyboards could be added to the same machine as volume increases.

Mr. Summerfield said that the sorter, as well as other automation developments in the Postal Service, will not endanger jobs of career employees because of the tremendous mail load of the American Postal Service. Already the United States has about two-thirds of the entire world mail volume and the U. S. volume is increasing further every year.

Mr. Eppert told newsmen at a press conference held in connection with the demonstration that the new sorting equipment was engineered for production in 9 months by a special team of electro-mechanical experts to meet rigid postal specifications.

In addition to the ten actual machines, Burroughs is furnishing ten training devices. Each of these training devices will have ten practice keyboards or total facilities for training 100 clerks. This training machine simulates sorting speeds encountered on the actual sorter and records the number of correct and incorrect sorts made by the trainee. The ten training devices are included in the total contract.

The Detroit post office, Mr. Summerfield pointed out, is the clearing house for all Detroit Metropolitan Area mail, and for nearly two-thirds of all the mail coming into Michigan. The Detroit office will move to an ultra-modern 23 million dollar building late next year which will be equipped with 30 of this type letter sorter.

The Burroughs sorter consists of several major components: operator control stations (consoles); encoding and decoding units; a re-circulating conveyor system; sorter bins; and a supervisor's control station.

Intermixed letters of varying sizes are loaded into a hopper at each operator console. A vacuum-head arm automatically selects the letters one at a time and places them on a scanning board in front of the operator, who reads the address and indexes the destination on a keyboard with 10 keys similar to those of a piano. The letter than drops into the conveyor system.

The conveyor, used to transport letters from the consoles to the proper sorting bins, consists of 161 letter carts —each divided into 12 compartments —that are constantly in motion. Shaft holding a series of nylon code wheels extends out from each compartment of the cart.

An electro-mechanical signal from the keyboard arranges the code wheels of the compartments so that when the wheels pass over a complementing code pattern at the destination bins they will release the letter into the appropriate sort bin. The code wheels then pass through an ingenious decoding unit and the process is repeated.

Only one pocket in each letter carrier is assigned to each operator control station, and the pockets carry only one letter at a time. That is, a letter processed through the first console drops into the first compartment of each carrier, a letter from the second console drops into the second pocket and so on. If the address on a letter is illegible, the operator sends it to a reject pocket simply by depressing all of the keys at once.

Speed of the conveyor system, and thus the sorting operation, is controlled by the supervisor. If operators are sorting 43,200 letters an hour, for example, the conveyor system moves at a speed of 21 inches per second.

The machine's 279 sorting bins are arranged in tiers and are easily accessible for emptying. An alarm system at the supervisor's control station signals when any of the bins are near capacity. Automatic counting devices also register the number of letters being processed according to operator control station. In addition, each letter is coded with a symbol identifying the operator control station where it entered the system.

The sorter will handle letters ranging from $2\frac{3}{4}'' \times 4''$ to $6'' \times 12''$ in size.

Microwave Data Processing.—A new method of transmitting scientific and business data by microwave is now being used by North American Aviation, Inc. Transmission is at the rate of 3000 words per second, 180,000 words every minute. The communication system links together six large-scale IBM computers at the company's Los Angeles Division, International Airport, and the Rocketdyne Division, 39 miles away in Canoga Park. Soon to be included in the data link will be North American's Missile and Autonetics Divisions, located in Downey, Calif., 18 miles from the Los Angeles plant and 40 miles from Rocketdyne.

This new microwave link will give North American a combined commercial computing capacity larger than any now existing at any one location. It will make the computers readily available to engineers in any of the divisions for high priority project studies as well as being used for payroll, inventory, test data and other uses.

It is the first time information has been transmitted from one computer to another without using a direct wire link. Engineers from Pacific Telephone and Telegraph Co., and the IBM Corporation's Advanced Systems Development Division, developed the system for North American.

The microwave link will make fuller use of the IBM computers in North American's scattered divisions and permit the company to end its use of slower couriers to take tape data back and forth from division to division.

The system also has great potential for the military services not only for the high rate of speed in which information can be transmitted, but its range. During development of the system, data was transmitted 2000 miles experimentally using spare television channels during off hours.

Zone Melting of Boron.—The discovery of a method for pressing rods of commercially available powdered boron has led in turn to successful floating zone melting of that extremely high-melting element. Developed at Bell Telephone Laboratories by Dr. E. S. Greiner, the technique will provide larger crystals for basic research studies of this semiconductor than have been available before.

Because of its physical properties, boron does not lend itself to floating zone melting, since it is not possible to form the powder into a suitable shape by pressure alone. The pressed forms are so friable that they crumble as soon as any attempt is made to remove them from the forming dies.

In one aspect of Dr. Greiner's new technique, boron powder, -20 to 100 mesh in size, is placed in boiling boric acid solution, and the mixture boiled to dryness. This coats the boron granules with boric acid, and the powder can then be pressed into forms which can be handled without breaking.

The formed bars, or compacts, are heated in two stages under vacuum for one hour, at temperatures of 300 to 600 C. to decompose the boric acid

into boron oxide. This compound forms a liquid coating on the boron particles. When the compact is cooled, this liquid hardens, and bonds the powder into a strong bar.

The bar of boron powder is mounted vertically for floating zone melting, and a high frequency induction coil is used to melt a narrow band or zone of the material. The molten zone is moved down the rod by raising the bar through the coil, while the melted boron freezes or crystallizes above the coil. During this operation, the boron oxide binder sublimes before the melting point of boron (about 2000 C.) is reached. Crystals as large as 0.1 in. were obtained, many of which were twinned.

The apparatus used in the floating zone refining is similar to that used in the treatment of other materials, except that the rod of boron is enclosed within two concentric transparent quartz tubes. The inner tube is slotted; when the subliming boron oxide coats the tubes enough to restrict vision, the slotted tube is rotated to give a clear field again. This modification to the equipment has previously been used in the floating zone melting of gallium arsenide at the Bell Laboratories by J. M. Whelan and G. H. Wheatley.

Preparatory to mounting the compact, the corners of the square bar are ground off, to minimize the "cage" effect, in which the corners do not melt. Also, in order to lower the resistivity of the boron, to allow it to be heated and melted by high frequency induction, the top end of the bar is mounted in a graphite cup, which can be readily heated by such means.

Nuclear Emulsion Recovery Vehicle.—America's newest entry in the race for space will be a probe vehicle which may be boosted 10,000 miles into deepest space and then recovered back on Earth. The National Aeronautics and Space Administration (NASA) has awarded General Electric Company's Missile and Space Vehicle Department, in Philadelphia, a $536,000 contract to develop a device for measuring the radiation belts surrounding the earth.

The device, called a nuclear emulsion recovery vehicle (NERV) is designed to obtain more complete measurements of the Van Allen Radiation Belts surrounding the earth. These measurements may help determine design factors involved in protecting man from radiation hazards in deep space flights.

Research and development of the cone-shaped, 75-lb. payload, about 19 in. in diameter and 18 in. long, will be conducted in the G. E. Missile and Space Vehicle Department plant in Philadelphia.

In initial launches next year, the NERV will be mounted on NASA's four-stage solid-propellant ARGO D-8 research rocket. This rocket will boost the vehicle to an altitude of about 1800 miles. In later flights, NASA's four-stage scout rocket may be used to loft the NERV to 10,000 miles altitude. Flights will probably be made from the Pacific Missile Range where the vehicle could be fired toward the south to coincide with the earth's magnetic lines of force.

The heart of the NERV experiment will be a cylindrical disc $\frac{1}{2}$ in. thick and 3 in. in diameter. The disc will consist of strips of radiation sensitive material called nuclear emulsion made by Ilford, Ltd. of England. During flight, the disc will be exposed to the radiation field. Exposure will be accomplished by means of an electro-mechanically operated shutter which will open at an altitude of about 200 miles, remain open to a height of about 1800

miles, and close when the vehicle has dropped to about 200 miles. During exposure, the disc will rotate behind a port in a shielded container.

The level and type of radiation encountered will be determined by examination of tracks made in the emulsion by radiation particles. Extensive laboratory analysis of the tracks should enable scientists to identify the particles and show the variations in the radiation field with altitude and latitude.

The nuclear emulsion recovery vehicle will be buoyant and will be equipped with a recovery system consisting of a silverized parachute, electronic search beacons, flashing lights, radar chaff, and a dye marker. The parachute will open at about 40,000 ft. to slow the vehicle for impact. In initial flights impact will be about 2000 miles from the launching area. Ships and planes will assist in the recovery operations.

An ablative material will be used to protect the vehicle from the heat generated during re-entry.

Air Reservists' Statistics Turned Over to Electronic Data-Processing System.—Vital statistics needed to maintain the U. S. Air Force "manpower bank" of more than 535,000 reservists at peak readiness were recently placed under the jet-speed control of an electronic data processing system.

"The computer will save us time—time that can be particularly valuable in the event of mobilization," said Brigadier General W. C. Morse, commander of the Air Reserve Record Center, as he placed in service the RCA 501, an all-transistor electronic data processing system developed by the Radio Corporation of America.

One of the largest personnel centers in the world, the ARRC is the clearing house for the records of all air reservists not on active duty. The center processes the files of 134,609 reserve officers and 400,993 airmen, a mass of documents that would stretch six miles end-to-end. Each reservist's file is a constantly updated record of his service, skills, capabilities and availability.

Established in 1954 in a consolidation of regional centers, the ARRC now processes an average of four and a half million pieces of correspondence a year, principally letters containing information to be added to individual records or requests for information on a reservist's status in various activities. With the RCA 501 system the vast ARRC library of data is stored on just ten reels of magnetic tape, each smaller than a landing wheel on a pre-World War I fighter plane.

While it formerly required 36 hours to extract the necessary information from the files to answer a reservist's query, the reply now will be on its way the same day.

"The manual system for pulling a reservist's record to extract a bit of information took 22 minutes," General Morse said. "Assuming 1000 such actions a day, the total job would consume 367 man hours of effort. While doing other functions simultaneously, the computer can do this same job in 17 minutes."

In addition to handling correspondence from reservists, the ARRC is required to have ready access to information on reserve strength, skills and availability for reports to Headquarters, U. S. Air Force, the Continental Air Command, Department of Defense and members of Congress.

The most important function of the ARRC, General Morse said, is maintaining the Air Force Reserve mobilization potential.

"This is where our computer system will be of great value," he added. "It will be used to identify reservists who possess the necessary skill and training

for assignment to mobilization positions. To assure that the Air Force mobilization plans are kept completely current, these jobs must be filled with properly qualified reservists."

To obtain this type of information, the ARRC distributes personnel surveys to reservists.

The RCA 501 system at the reserve center will perform a number of other important tasks. In the Air Force annual assessment of officers for promotion, the computer will pull from its memory a list of officers eligible for consideration by grade break-down. If an individual meets all requirements and is favorably passed upon by a review board, his eventual promotion will be entered in his magnetic tape record file by the RCA 501.

The data processing system also will assist in the preparation of labels to mail 330,000 copies of the Air Reservist Magazine monthly and 20,000 copies quarterly of the Medical Reservist Bulletin.

Queries from such agencies as the Veterans Administration will be processed on a same-day basis.

"Inquiries from other agencies or from individuals may directly affect the reservist's private life," General Morse said. "For instance, a man leaving active duty may want to buy a new home. He applies to the Veterans Administration and that agency writes us for information on his background and military record. Fast action is important."

New Facts on Solar X-Rays.—New facts about X-rays from the sun which penetrate into the Earth's upper atmosphere were obtained in the recently-completed Project Sunflare II. The solar flare study, conducted by the U. S. Naval Research Laboratory, Washington, D. C., determined the existence of X-rays tens of thousands of volts greater than detected in any previous experiments.

The project was a sequel to two previous NRL solar flare studies with rockets—Project Rockoon, begun 1956, and its continuation in 1957 for the International Geophysical Year, Sunflare I. The recent experiments produced new information as a result of equipping the high altitude rockets with instruments which would extend the coverage of wavelengths to include emissions from a quiet or moderately active sun as well as during flares.

The most important information gained by the project was the discovery of X-rays produced in the most active phases of flares. X-rays with energies as high as 80,000 volts were detected above the absorbing atmosphere of the Earth. The findings indicate that temperatures in the solar atmosphere may be as high as one hundred million degrees centigrade.

The measurements of energetic X-rays were accomplished with scintillation counters of very high sensitivity. It is of interest to note that the Russians instrumented Sputnik III with an elaborate experiment to search for such very energetic X-ray emissions from the sun. They reported that on the 6th of June 1958, during the occurrence of a group of strong solar flares, no X-ray flux was detected; also, when the satellite passed from night to day there was no indication of radiation from the quiet sun. The reason the Russian scientists did not observe these emissions was that the sensitivity of their equipment was not great enough to measure fluxes of the magnitudes observed in the NRL rocket experiments.

Measurements first made in Project Rockoon had indicated that there was a background of X-rays at high altitudes with energies extending to perhaps as high as 50,000 volts in

energy. The flux was so small that it was not possible in that series of experiments to make directional measurements which could identify the radiation with the sun. It was theoretically possible that such X-ray emission could originate in some celestial source or possibly as a secondary effect of the interaction of Van Allen belt electrons with the atmosphere. The results of Project Sunflare II indicate that such X-rays are a normal solar emission. It appears that even the quiet sun emits a broad spectrum of X-rays extending to very high energies, but that the flux is very low. The intense excitation that accompanies a solar flare enhances the X-ray emission over the entire spectral range.

NRL used 12 Nike-Asp rockets for the Sunflare II experiments, with instrumentation payloads of about 55 lb. Eight of the twelve rockets were completely successful, attaining altitudes between 130 and 150 miles, with approximately 8 minutes of flight data continuously telemetered during the best flights. Three of these were launched during flares and five during solar quiescence and minor activity.

The project was conducted for the International Geophysical Cooperation 1959 and was supported by a grant from the National Science Foundation.

YOU CAN ADVANCE SCIENCE EDUCATION

Today, more than ever before in its 135-year history, there is vital need for The Franklin Institute to effectively promote education in science and technology. It is imperative that we meet this challenge by providing adequate educational opportunities in these fields. This requires vision, objective planning, and money. We have more than enough of the first two requisites, but far too little of the third.

Our programs are aimed at professional scientists and industry, as well as the lay public and young people seeking inspiration and guidance in choosing a career. The Institute's educational programs are impressive, for they begin with students in the early grades of our elementary schools and continue throughout an individual's professional or industrial life. With more funds at our disposal, the scope and vigor of these activities could be greatly increased.

The Franklin Institute is not richly endowed. It is a non-profit organization, depending for encouragement and support on an understanding public. Capable and conservative management assures wise administration of all funds.

Your gift or bequest, large or small, will be deeply appreciated and will be used effectively to broaden the Institute's educational usefulness. There is a warm satisfaction in giving financial support to an organization that has pioneered in, and is dedicated to, the advancement of science and technology through education.

When property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

The Secretary of The Franklin Institute will gladly furnish you with additional information. Write to him at The Franklin Institute, Benjamin Franklin Parkway at Twentieth Street, Philadelphia 3, Pennsylvania.

"Magic Paper" for Guarding Secrecy of Facsimile Mail.—Alden Electronic & Impulse Recording Equipment Co., Inc. of Westboro, Mass., recently demonstrated to Post Office and International Telephone & Telegraph engineers a new facsimile recording paper from which the message disappears immediately after recording to reappear at a given time later in all of its original fidelity.

According to Milton Alden, chairman of Alden Electronic, the new paper can eliminate many of the security precautions now visualized as necessary in order to maintain privacy of the mails at the receiving end. "This startling development has tremendous implications in solving the basic problem of maintaining the secrecy of facsimile mail that may be sent by the Post Office," he said. Heretofore, it has appeared that complicated paper handling equipment would be necessary, Mr. Alden pointed out.

Called the "magic paper" by an ITT engineer who viewed the demonstration, it allows transmissions to be received directly on the recorder, then processed into envelopes prior to the message reappearing, so that only the original sender and the recipient would see the message. It not only is seen as a great boon to the Post Office modernization program for maintaining sanctity of mail sent by facsimile, but also for private business facsimile systems, where it can be used to send legal and confidential documents. It is regarded as being of utmost importance to military operations for safeguarding classified communications.

The disclosure of the "magic paper" came as part of a demonstration of Alden facsimile equipment which included large size, flat-copy scanners and transmitters for sending two letters at a time, (at an approximate rate of two letters per minute) drawings, advertising layouts and other business and communication data, without regard to size of the original document. This equipment is now in operation for the U. S. Weather Bureau's new high-altitude facsimile weather map network, which also uses Alden automatic continuous facsimile recorders.

The techniques shown are said to represent a breakthrough from the conventional slow speeds now used for business facsimile and for ordinary weather map transmission. They permit transmission speeds from 10 to 15 times faster than those presently commercially available and may provide the answer for high volume facsimile traffic over the high quality lines now becoming more generally available. They are more practical from the standpoints of equipment size, reliability, ease of operation by office personnel and cost than would be the extremely complex sophisticated equipment and facilities required for ultra-high-speed facsimile.

The "magic paper" demonstrated was made available to Alden Electronic & Impulse Recording Equipment Co., Inc. by the Alfax Paper and Engineering Company, also of Westboro. It has come out of their continuing research and development of electrosensitive facsimile papers which have the speed, sensitivity and stability for operation with high speed helix recorders and which eliminate the high current "burning" of a coated paper surface, using conventional, stylus-type recorders.

THE FRANKLIN INSTITUTE
LABORATORIES FOR RESEARCH AND DEVELOPMENT

announce opportunities for:

ELECTRICAL ENGINEERS

SYSTEMS ENGINEERS

ORGANIC CHEMISTS

OPERATIONS RESEARCH SPECIALISTS

MECHANICAL ENGINEERS

EXPERIMENTAL PSYCHOLOGISTS

METALLURGISTS

INDUSTRIAL ENGINEERS

PHYSICISTS

APPLIED MATHEMATICIANS

to work on challenging problems in research

Send complete resume to:

Mr. John E. Christ, *Director of Personnel*
THE FRANKLIN INSTITUTE
Philadelphia 3, Pa.

THE FRANKLIN INSTITUTE LABORATORIES

FOR RESEARCH AND DEVELOPMENT

In the fields of engineering and the physical sciences, a competent and versatile staff of several hundred scientists and engineers, working with modern equipment in a creative climate, is trained to bring a fresh scientific approach to the solution of industrial problems.

THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA

BENJAMIN FRANKLIN PARKWAY AT 20TH ST. PHILADELPHIA 3, PA.